

**BỘ GIÁO DỤC VÀ ĐÀO TẠO
TRƯỜNG ĐẠI HỌC CÔNG NGHỆ VÀ KỸ THUẬT TP HCM**

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< CÁC BÀI BÁO ĐÃ CÔNG BỐ CÓ LIÊN QUAN ĐẾN ĐỀ TÀI >

TP HCM, /2026

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Dưới đây là danh mục các bài báo khoa học về kết quả nghiên cứu của luận án đã
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Stt	Tên bài báo	Tác giả	Tên tạp chí/kỷ yếu, số/tập, số ISSN/IS BN	Thời gian xuất bản	Link/Doi	Ghi chú
I.	Tạp chí khoa học trong danh mục WoS (SCIE, SSCI, AHCI, ESCI)					
1	Determining with High Accuracy the Relaxation Modulus and Creep Compliance of Asphaltic Materials in the Form of Sums of Exponential Functions from Mathematical Master Curves of Dynamic Modulus;	Thanh-Nhan Phan; Duy-Liem Nguyen; Tan-Van Vu; Vu-Tu Tran; H. T. Tai Nguyen; and Mai Lan Nguyen	0899-1561	6/2022	10.1061/(ASCE)MT.1943-5533.0004366.	
2	Development of a generalised creep-recovery test and a back-calculation method for determining the permanent deformation of asphalt mixtures in the time domain	Vu-Tu Tran, Thanh-Nhan Phan, Van-Tieng Tran, Tien-Tho Do, H. T. Tai Nguyen*,	2164-7402	2/2023	10.1080/14680629.2023.2180291	

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3	Verification of time-temperature superposition principle for viscoplastic strains of asphalt mixtures obtained from creep-recovery tests in the low- strain regime	Thanh- Nhan Phan, Ngoc- Hung Nguyen, Tien Tho Do, H.T. Tai Nguyen*, Mai Lan Nguyen	2164- 7402	8/2024	10.1080/14680629.2024.2390156	
II	Tạp chí khoa học trong danh mục Scopus					
III	Tạp chí khoa học quốc tế có ISSN, có phản biện nhưng chưa xếp hạng WoS/Sopus					
IV	Kỷ yếu hội nghị khoa học quốc tế có phản biện và có ISBN					
1	Characterizing The Creep Compliance of A Conventional Asphalt Mixture Using Creep- Recovery Test;	Thanh- Nhan Phan, H.T. Tai Nguyen	978-3- 031- 96122-9	10/202 5	https://doi.org/10.1007/978-3-031-96122-9_18	
V	Kỷ yếu hội nghị khoa học trong nước có phản biện và có ISBN					
1	So sánh độ chính xác của các phương pháp xấp xỉ bằng tổng của hàm số mũ sử dụng các hệ số có giá trị thực	Phan Thành Nhân, Nguyễn Huỳnh Tấn Tài	978-604- 357-085- 4	12/202 2		
2	Evaluating the accuracy of linear viscoelastic properties in the time domain analytically approximated from complex modulus data	Nguyễn Ngọc Hung, Phan Thành Nhân, Đỗ Tiến Thọ, Nguyễn Huỳnh Tấn Tài;	976-604- 357-277- 3	4/2024		

Determining with High Accuracy the Relaxation Modulus and Creep Compliance of Asphaltic Materials in the Form of Sums of Exponential Functions from Mathematical Master Curves of Dynamic Modulus

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H. T. Tai Nguyen⁵; and Mai Lan Nguyen⁶

Abstract: According to the physical causality principle, the two components of complex modulus are not independent yet interrelated to each other via Kramers-Kronig relationships. Using two mathematical functions to predict simultaneously the dynamic modulus (DM) and phase angle (PA) generally leads to the violation of the causality principle. Therefore, the calculation of one component based on the other one is a vital task for using mathematical models to predict the linear viscoelastic (LVE) behavior of asphaltic materials. In addition, the relaxation modulus (RM) or the creep compliance (CC) in the form of sums of exponential functions (SEFs) is necessary for the efficient resolution of the stress-strain relationship in the time domain. This paper presented a novel method for calculating with high accuracy the RM and CC in the form of SEFs based on the input DM master curve. First, the midpoint integration scheme was used to calculate the PA based on the exact relationship between the PA and DM. After that, the storage modulus and compliance were computed and decomposed with high accuracy into SEFs using a special technique. Then, the closed-form formula of the RM and CC were derived thanks to the special form function of the storage modulus and compliance. Finally, the special technique was used once again to decompose with high accuracy the RM and CC into SEFs for possible uses in finite-element method (FEM) analysis in the time domain. The results showed that the relative errors of all LVE properties computed were lower than 0.1 percent at all calculating points. DOI: [10.1061/\(ASCE\)MT.1943-5533.0004366](https://doi.org/10.1061/(ASCE)MT.1943-5533.0004366). © 2022 American Society of Civil Engineers.

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Introduction

Relaxation modulus (RM) and creep compliance (CC) are fundamental properties characterizing the linear viscoelastic (LVE) behavior of asphaltic materials like asphalt binder and mixture, based on which the stress response of the material subjected to an

input strain history and vice versa can be established (Tschoegl 1989). An accurate determination of the RM and CC will contribute to a better characterizing and analysis of damage, cracking, and rutting of asphaltic materials. It has been widely accepted that the stress and strain in the time domain can be represented by the complex stress and strain. The complex modulus and compliance are therefore defined as the ratio of the complex stress to the complex strain and the inverse of that ratio (Cho 2016). Therefore, the LVE behavior of an asphaltic material can be characterized by either the RM/CC in the time domain or the complex modulus/compliance in the frequency domain. All these LVE properties are equivalent to each other because of the unicity of the behavior of the material.

In practice, it is difficult to experimentally determine the RM and CC using quasi-static testing because of the viscoplastic strain that always exists in the measured strain at intermediate and high temperatures. However, the viscoplastic strain does not affect the measurement of complex modulus/compliance where cyclic loading is applied (Nguyen et al. 2021a). Therefore, complex modulus/compliance test procedures have widely been used by practitioners and researchers to characterize the LVE behavior of asphaltic materials (e.g., Olard and Di Benedetto 2003; Nguyen et al. 2013, 2021b; Pellinen et al. 2003; Rowe et al. 2009; Liu and Luo 2017; Tan et al. 2020). In such test procedures, both two components of the complex modulus/compliance in the polar plane can be determined, based on which the two components in the complex plane can be calculated.

Because there is only one relationship between the complex modulus and the RM and between the complex compliance and

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CC, the two components of the complex modulus and compliance cannot be independent (Booij and Thoone 1982). They are interrelated to each other via Kramers-Kronig relations for the physical causality principle to be satisfied (Booij and Thoone 1982; Tschoegl 1989). Therefore, knowing only one component of the complex modulus/compliance in the entire range of frequency allows understanding the full LVE behavior of the material. Thus, the master curve of the LVE properties should be carefully constructed to satisfy the Kramers-Kronig relations. This is a challenging task for mathematical master curve models. The reason is that a mathematical equation can only predict one single component of the complex modulus/compliance at a time, and an explicit equation for the interrelationship between the two components cannot be exactly prescribed.

Among mathematical models for LVE behavior of asphaltic materials, sigmoid function-based models have been the most widely used in the asphalt community for their simplicity and excellent fitting of the dynamic modulus (DM) (Kim et al. 2015; Liu and Luo 2017; Liu et al. 2018; Pellinen et al. 2003; Rowe 2009; Rowe et al. 2009; Tan et al. 2020; Xi and Luo 2021). The parameters of the sigmoid functions can be correlated to the characteristics of asphalt mixtures and the binder used in the mixture to construct generic formulas of DM (Andrei et al. 1999; Bari 2005). Although the generic formulas of DM were not as accurate as artificial neural network (ANN)-based predictions as demonstrated by Ceylan et al. (2009), they have been used as core formulas in the *Mechanistic-Empirical Design Guide* software (MEPDG 2004) for their explicit form function and good predictive ability for various types of asphalt mixtures.

Several techniques have been proposed to ensure a weak or strong satisfaction of Kramers-Kronig relations for the sigmoid function-based models. In an early study of Liu and Luo (2017), the phase angle (PA) master curve was first derived and then used for constructing the fitting error function involving both the modified sigmoid (MS) function of DM and the derived function of PA. Similarly, the function of loss modulus was derived from the MS function of storage modulus, and both functions were used in the construction of the fitting error function. The analytical approximate formulas of Booij and Thoone (1982) were used to derive the master curves of PA and loss modulus from those of DM and storage modulus, respectively. With this approach of Liu and Luo (2017), the Kramers-Kronig relations were weakly ensured, which may remedy the deficiency of using two different mathematical functions to simultaneously predict the DM and PA data (e.g., Naik and Biligiri 2015; Rahman and Tarefder 2018).

The approach was then extended by the same research group (Liu et al. 2018) to calculate the RM master curve. The exact formula for deriving the continuous relaxation spectrum from the MS function of storage modulus was proposed, based on which the RM was numerically calculated. Lack of consistency between LVE properties may still exist because Kramers-Kronig approximate formulas were involved in the formulation. More recently, Xi and Luo (2021) proposed numerical calculation of the PAs and loss moduli to strongly ensure the physical causality. However, a numerical method for calculating the RM and CC from the master curve of dynamic or storage modulus has not yet been made available by the group of authors.

The numerical calculation of PA from the DM data can be dated back to the work of Parot and Duperray (2007), in which a numerical algorithm was proposed to deal with the scattered data of DM. Without this treatment, spurious waviness of phase angles computed from experimental DM data may occur (Rouleau et al. 2013). However, basic integration schemes such as trapezoidal and midpoint rules can be applied to accurately calculate the PA from the predicted master curve of DM or the loss modulus from the predicted master curve of storage modulus (e.g., Xi and Luo 2021). Furthermore,

several researchers have reported considerable difficulties when measuring the PA, especially at low frequency (e.g., Rowe 2009; Zhang et al. 2020). The problem seems to stem from the extremely small values of the time lag between the stress and strain signals as well as oscillations of the measured signals in the dynamic testing. Thus, in case the accuracy of the PA data is not as good as that of the DM data, only the master curve of the DM should be constructed and predicted by a mathematical model. After that, the master curve of PA must be accurately calculated from the DM master curve. With this process, the large discrepancy of the PAs tested is expected to not affect the overall R^2 value of the fitting process.

In terms of time-frequency interconversion for LVE properties, a large number of numerical methods have been proposed in the literature, which can be shortly summarized as follows. Baumgaertel and Winter (1989) have proposed the fitting of the storage and loss modulus by functions equivalent to those of the generalized Maxwell (GM) model with the aid of an undisclosed nonlinear least-square program. The approach was then modified by Mead (1994). Instead of determining the relaxation times, they were selected a priori to turn the nonlinear least-square optimization into an easier linear problem. Later, Mun et al. (2007) proposed to fit only the storage modulus/compliance by that of the GM model and to adjust the PA values before the fitting process to improve the accuracy of the fitting. A predefined set of relaxation/retardation times was also selected to prevent a nonlinear resolution. More recently, Zhang et al. (2018) used the GM model to predict both the storage and loss modulus calculated from the MS function of DM and the pre-smoothed PA function of Rahman and Tarefder (2018). However, the satisfaction of Kramers-Kronig relations in the formulation was left undiscussed by those authors.

A detailed discussion on analytical approximate formulas for time-frequency interconversion has been presented by Schapery and Park (1999). Owing to the simplicity and good accuracy of the analytical approximate formulas of Schapery and Park (1999), they have been widely used in the literature. Emri et al. (2005) have shown that most analytical approximate formulas, including those of Schapery and Park (1999), achieved an acceptable level of accuracy (lower than 5%). The same level of accuracy was also reported by Nguyen et al. (2021a, 2022) when dealing with the Huet-Sayegh (HS) model. Although the analytical approximate formulas are useful in calculating the values of RM and CC, their form functions are not suitable to develop a recursive formula of stress and strain, with which the computational efficiency will be significantly improved.

Therefore, this paper aimed at developing a method for determining with high accuracy (lower than 0.1%) the RM and CC from the mathematical master curve of DM. The sought formulation must fully obey the physical causality principle and LVE theory. In addition, the RM and CC must be decomposed with high accuracy into the sums of exponential functions (SEFs), or Prony series, for a computational efficiency of finite-element method (FEM) analysis of asphaltic structures in the time domain.

Theoretical Formulation

In what follows, classical equations of LVE theory will be summarized, based on which the new formulation will be presented for calculating with high accuracy the RM and CC from mathematical DM master curves.

Fundamental LVE Properties of Asphaltic Materials

Let $E(t)$ denote the LVE relaxation modulus, which describes the stress response of a viscoelastic material subjected to a unit step

function of strain. Knowing $E(t)$ in the entire range of time allows understanding the full LVE behavior of the material. In effect, assuming that the strain is small enough such that the Boltzmann superposition principle still holds, the stress in the material caused by an arbitrary strain history is given as follows (Tschoegl 1989):

$$\sigma(t) = \int_{-\infty}^t E(t-\tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau \quad (1)$$

where $\varepsilon(t)$ = strain history; and $\sigma(t)$ = stress response. Let us now study a harmonic strain input $\varepsilon(t) = \varepsilon_0 \cos(\omega t)$, where ε_0 is the magnitude and ω is the angular frequency of the signal. Assuming that the previous strain input signal is the real part of the complex strain $\varepsilon(i\omega t) = \varepsilon_0(\cos(\omega t) + i \cdot \sin(\omega t))$, we have

$$\sigma(t) = \Re \left[\varepsilon_0 i\omega \int_{-\infty}^t E(t-\tau) \cdot e^{i\omega\tau} \cdot d\tau \right] \quad (2)$$

where $i^2 = -1$ is the imaginary unit; $\Re(z)$ = real part of z ; and the equality $e^{i\omega t} = \cos(\omega t) + i \cdot \sin(\omega t)$ has been used. Performing the change of variable $u = t - \tau$ in Eq. (2) gives rise to

$$\sigma(t) = \Re \left[\varepsilon_0 \cdot e^{i\omega t} i\omega \int_0^{+\infty} E(u) e^{-i\omega u} du \right] \quad (3)$$

By convention, one defines the complex modulus as follows (Tschoegl 1989):

$$E^*(i\omega) = i\omega \int_0^{+\infty} E(t) e^{-i\omega t} dt \quad (4)$$

Then, the real-valued stress response of the material caused by a harmonic strain input is, therefore, the real part of the complex stress $\sigma(i\omega t)$

$$\sigma(t) = \Re[\sigma(i\omega t)] = \Re[E^*(i\omega) \cdot \varepsilon(i\omega t)] \quad (5)$$

The real and imaginary parts of the complex modulus are called storage (E') and loss moduli (E''), and its magnitude and argument are called dynamic modulus ($|E^*|$) and phase angle (φ_E). Similarly, the corresponding complex compliance is defined by convention (Tschoegl 1989) as follows:

$$D^*(i\omega) = i\omega \int_0^{+\infty} D(t) e^{-i\omega t} dt \quad (6)$$

where $D(t)$ = CC representing the strain response of the viscoelastic material caused by a unit step function of the stress. It can be easily demonstrated that (Tschoegl 1989)

$$\begin{aligned} D'(\omega) &= |D^*(i\omega)| \cos(-\varphi_E(\omega)); & D''(\omega) \\ &= |D^*(i\omega)| \sin(-\varphi_E(\omega)) \end{aligned} \quad (7)$$

where D' and D'' = real and imaginary parts of $D^*(i\omega)$, widely called storage and loss compliances. It can be seen in Eq. (7) that the PA of the complex compliance is the opposite of that of the complex modulus.

Kramers-Kronig Relationships

The two components of the complex modulus and compliance must obey Kramers-Kronig relationships for the physical causality principle to be satisfied (Booij and Thoone 1982) as follows:

$$\begin{aligned} E'(\omega) &= E^*(\omega \rightarrow \infty) + \frac{2}{\pi} \int_0^{+\infty} \frac{u E''(u)}{\omega^2 - u^2} du; \\ E''(\omega) &= \frac{2\omega}{\pi} \int_0^{+\infty} \frac{E'(u)}{u^2 - \omega^2} du \end{aligned} \quad (8)$$

$$\begin{aligned} \ln |E^*(\omega)| &= \ln |E^*(\omega \rightarrow \infty)| + \frac{2}{\pi} \int_0^{+\infty} \frac{u \varphi_E(u)}{\omega^2 - u^2} du; \\ \varphi_E(\omega) &= \frac{2\omega}{\pi} \int_0^{+\infty} \frac{\ln |E^*(u)|}{u^2 - \omega^2} du \end{aligned} \quad (9)$$

Because their integral is equal to zero, $\omega \varphi_E(\omega)/(\omega^2 - u^2)$ and $\ln |E^*(\omega)|/(\omega^2 - \omega^2)$ were omitted in Eq. (9). Kramers-Kronig equations for complex compliance are obtained straightforwardly by substituting the character E in Eqs. (8) and (9) by character D and are not presented here. In practice, the PA is more difficult to be accurately measured than the DM, as previously discussed. Therefore, the analytical approximate formula of PA proposed by Booij and Thoone (1982) was chosen to be presented here

$$\varphi_E(\omega) = \frac{1}{\pi} \int_0^{+\infty} \ln \left| \frac{u + \omega}{u - \omega} \right| \frac{d \ln |E^*(u)|}{d \ln u} d \ln u \quad (10a)$$

$$\varphi_E(\omega) \simeq \frac{\pi}{2} \left(\frac{d \ln |E^*(u)|}{d \ln u} \right)_{u=\omega} \quad (10b)$$

To obtain this approximation, the rate of change of the derivative of $\ln |E^*(u)|$ with respect to $\ln u$ was assumed to be much smaller than that of $\ln |(u + \omega)/(u - \omega)|$ so that the derivative can be put outside the integral, and the following equality has been used:

$$\int_0^{+\infty} \ln \left| \frac{u + \omega}{u - \omega} \right| d \ln u = \int_{-\infty}^{+\infty} \frac{\ln |v|}{v^2 - 1} dv = \frac{\pi^2}{2} \quad (11)$$

Because the increase rate of the derivative is not always small compared with the rate of change in value of $\ln |(u + \omega)/(u - \omega)|$, the accuracy of this approximation is not very good, as will be shown in the next section.

If the DM data have been predicted (or fitted) using a mathematical model, Eq. (10a) can be numerically integrated as follows. Let the range of frequency of interest be discretized by $2N + 1$ equidistant points in the base 10 logarithmic scale

$$\log \omega_i = \{\log \omega_{-N}, \dots, \log \omega_0, \dots, \log \omega_N\}, \quad i = -N, \dots, N \quad (12)$$

The dummy variable u in Eq. (10a) was also discretized by using $2M + 1$ points in the base 10 logarithmic scale equally spaced at a step of $h = (\log \omega_M - \log \omega_{-M})/(2M)$. By using the classical midpoint rule, the PA in Eq. (10a) can be calculated

$$\begin{aligned} \varphi_E(\omega_i) |_{i=-N, \dots, N} &\simeq \frac{h}{\pi} \cdot \sum_{j=1}^{2M-1} \ln \left| \frac{10^{\log u_{j+1/2}} + 10^{\log \omega_i}}{10^{\log u_{j+1/2}} - 10^{\log \omega_i}} \right| \\ &\cdot \left(\frac{d \ln |E^*(u)|}{d \ln u} \right)_{u=u_{j+1/2}} \end{aligned} \quad (13)$$

It should be mentioned that the integral in Eq. (10a) must be considered as the principal part because of the singular points $u = \pm \omega$ (Lakes 1999). Therefore, the points $\log u_{j+1/2} = \pm \log \omega_j$, if any, will be ignored in the calculation of Eq. (13).

Time-Frequency Interconversion

The CC or RM function is required for defining the stress-strain relationship in the time domain. However, physical relationships do not exist for mathematical LVE models. Thus, the integral equations in Eqs. (4) and (6) were used to derive the LVE properties in the time domain.

RM and CC as Inverse Fourier Transform of Dynamic Viscosity

It can be demonstrated that (Cho 2016)

$$\frac{E^*(i\omega)}{i\omega} = \int_0^{+\infty} E(t) \cdot \cos(\omega t) \cdot dt - i \int_0^{+\infty} E(t) \cdot \sin(\omega t) \cdot dt$$

$$= \eta'(\omega) - i \cdot \eta''(\omega) \quad (14)$$

where $\eta'(\omega)$ and $-\eta''(\omega)$ = real and imaginary part of the complex number $E^*(i\omega)/i\omega$, called dynamic viscosity.

Remark 1: The real part of $E^*(i\omega)/(i\omega)$ is even when the imaginary part of $E^*(i\omega)/(i\omega)$ is odd.

The RM function can be obtained by performing the inverse Fourier transform

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{E''(\omega) - iE'(\omega)}{\omega} (\cos(\omega t) + i \sin(\omega t)) d\omega \quad (15)$$

where $E^*(i\omega)/(i\omega) = (E'' - iE')/\omega$ has been used. According to Remark 1, E''/ω is even, $-E'/\omega$ is odd; both $E'' \cos(\omega t)/\omega$ and $E' \sin(\omega t)/\omega$ are even with ω . It can be deduced that

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{E''(\omega) \cos(\omega t)}{\omega} d\omega + \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{E'(\omega) \sin(\omega t)}{\omega} d\omega$$

$$= \frac{1}{\pi} \int_0^{+\infty} \frac{E''(\omega) \cos(\omega t)}{\omega} d\omega + \frac{1}{\pi} \int_0^{+\infty} \frac{E'(\omega) \sin(\omega t)}{\omega} d\omega \quad (16)$$

which is a real-valued function. In addition, $E(t) = 0$ for t varying from $-\infty$ to 0. Because $E'' \cos(\omega t)/\omega$ is even whereas $E' \sin(\omega t)/\omega$ is odd with t , we have (Lakes 1999)

$$\int_0^{+\infty} \frac{E''(\omega) \cos(\omega t)}{\omega} d\omega = \int_0^{+\infty} \frac{E'(\omega) \sin(\omega t)}{\omega} d\omega, \quad \forall t \geq 0 \quad (17)$$

Hence, the equation for calculating the RM is obtained (Lakes 1999) as follows:

$$E(t) = \frac{2}{\pi} \int_0^{+\infty} \frac{E'(\omega) \sin(\omega t)}{\omega} d\omega$$

$$= \frac{2}{\pi} \int_0^{+\infty} \frac{E''(\omega) \cos(\omega t)}{\omega} d\omega, \quad \forall t \geq 0 \quad (18)$$

And by similarity

$$D(t) = \frac{2}{\pi} \int_0^{+\infty} \frac{D'(\omega) \sin(\omega t)}{\omega} d\omega$$

$$= \frac{2}{\pi} \int_0^{+\infty} \frac{D''(\omega) \cos(\omega t)}{\omega} d\omega, \quad \forall t \geq 0 \quad (19)$$

Development of a Closed-Form Formula for the RM and CC

Due to the complexity of Kramers-Kronig relationships, the closed-form formula of the storage and loss modulus/compliance cannot be determined from the master curve of DM and compliance. In addition, the time-frequency interconversion formulas are in the form of indefinite integral whose value cannot be analytically calculated in a general manner. Therefore, in order to derive a

closed-form formula of the RM and CC, the storage and loss modulus/compliance function must be approximated by some special functions for the indefinite integral to be analytically integrated. Because the equalities in Eqs. (18) and (19) are equivalent, we will consider the first equality involving only the storage modulus and compliance. It is obvious that the storage modulus increases with an increase of frequency but the storage compliance decreases, and that both functions tend to their asymptotes when $\omega \rightarrow \infty$. As a result, the storage modulus and compliance can be decomposed into sums of decaying functions like the exponential function. More importantly, the integral $\int_0^{+\infty} (e^{\alpha\omega} \cdot \sin(\omega t))/\omega d\omega$ can be analytically calculated in case $\alpha < 0$. Therefore, the remaining task is to decompose the storage modulus and compliance into SEFs as follows:

$$E'(\omega) \approx E^*(\omega \rightarrow \infty) - \sum_{j=1}^{N_j} E'_j e^{b_j \omega};$$

$$D'(\omega) \approx D^*(\omega \rightarrow \infty) + \sum_{j=1}^{N_j} D'_j e^{c_j \omega} \quad (20)$$

where b_j and $c_j < 0$ = real-valued negative decay rates; E_j and D_j = real-valued coefficients; and N_j = number of exponential functions used. The coefficients are not limited to be positive or negative numbers because Eq. (20) is just a mathematical approximation without any physical meaning. The larger the number N_j is, the higher the accuracy of the approximation that will be gained. Thus, the value of N_j must be chosen depending on the desired level of accuracy. Once the storage modulus and compliance have been approximated using SEFs, the closed-form formulas of the RM and CC are obtained

$$E(t) = E^*(\omega \rightarrow \infty) - \frac{2}{\pi} \sum_{j=1}^{N_j} E'_j \cdot \tan^{-1} \left(\frac{t}{-b_j} \right);$$

$$D(t) = D^*(\omega \rightarrow \infty) + \frac{2}{\pi} \sum_{j=1}^{N_j} D'_j \cdot \tan^{-1} \left(\frac{t}{-c_j} \right) \quad (21)$$

The proof of Eq. (21) is left in Appendix I for the sake of brevity. Once the RM and CC are obtained, they can be decomposed into

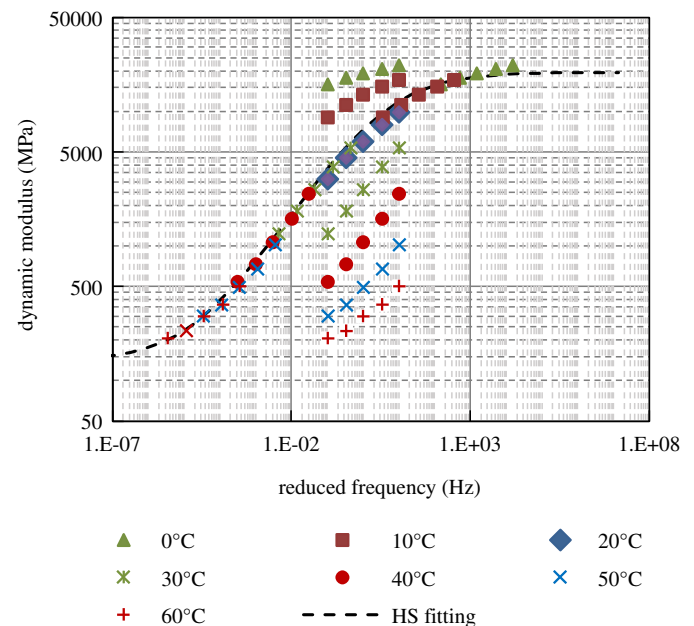


Fig. 1. DM master curve data of the unmodified asphalt mixture studied by Nguyen et al. (2021a).

SEFs for improving the computational efficiency of FEM analysis in the time domain. The summary of the proposed method in the form of an algorithm is presented in Algorithm 1 to facilitate the reading of this paper and to highlight the novelties in the proposed approach. The special technique used for approximating a decaying function by SEFs is the modified version of the Dombi algorithm presented by Nguyen et al. (2022), which will be shortly presented in the next paragraph.

Algorithm 1. Algorithm for determining the RM and CC in the form of SEFs from DM master curves

Input data: presmoothed (mathematical or mechanical) master curves of DM data:

- Define the range of frequency of interest (ω_i) in the logarithmic scale.
 - Define the range of time of interest (t_i) in the logarithmic scale.
- Step 1. Numerically calculate the PA $\varphi_E(\omega_i)$ using Eq. (13), then calculate the storage modulus and compliance

$$E'(\omega_i) = |E^*(i\omega_i)| \cdot \cos(\varphi_E(\omega_i));$$

$$D'(\omega_i) = \cos(\varphi_E(\omega_i)) / |E^*(i\omega_i)|$$

Step 2. Decompose with high accuracy the storage modulus and compliance into SEFs

$$E'(\omega_i) \approx E^*(\omega \rightarrow \infty) - \sum_{j=1}^{N_j} E'_j e^{b_j \omega_i};$$

$$D'(\omega_i) \approx D^*(\omega \rightarrow \infty) + \sum_{j=1}^{N_j} D'_j e^{c_j \omega_i}$$

Step 3. Calculate the RM and CC

$$E(t_i) = E^*(\omega \rightarrow \infty) - \frac{2}{\pi} \sum_{j=1}^{N_j} E'_j \cdot \tan^{-1}(-t_i/b_j)$$

$$D(t_i) = D^*(\omega \rightarrow \infty) + \frac{2}{\pi} \sum_{j=1}^{N_j} D'_j \cdot \tan^{-1}(-t_i/c_j)$$

Step 4. Decompose with high accuracy the RM and CC into SEFs for possible uses in FEM analysis in the time domain

$$E(t_i) = E(t \rightarrow 0) + \sum_{j=1}^{N'_j} E_j e^{b'_j t_i};$$

$$D(t_i) = D(t \rightarrow \infty) - \sum_{j=1}^{N'_j} D_j e^{c'_j t_i}$$

Approximation of a Decaying Function by SEFs Using the Modified Dombi Algorithm

It remains to us to deal with the approximation of the storage modulus and compliance into SEFs. Thanks to the work of Dombi (2005) and Nguyen et al. (2022), this kind of approximation can be performed with a controllable and high level of accuracy. The

algorithm mainly includes two steps. First, appropriate decay rates were determined based on an m -order polynomial whose coefficients are determined based on the values of the input function. Then, the coefficients of exponential functions were optimized to best fit the input data using linear least-square techniques. The algorithm is summarized in Appendix II for facilitating the reading of this paper. Interested readers are also invited to refer to Nguyen et al. (2022) and references quoted therein for a more detailed presentation of the algorithm.

Application Examples

This section presents application examples, which were carried out based on the previous algorithm to assess the efficiency of the proposed formulation. All the results were obtained with the aid of the MATLAB version R2015 program running on a Core i5 computer. The obtained results will be compared with either exact solutions, analytical approximate solutions, or experimental results for validation purposes. In the following, Eq. (22a) will be used to measure the relative error or (difference) of the computed results at all calculating points except Eq. (22b), which is devoted to evaluating the relative error or (difference) of the storage or creep compliance

$$\text{Err} = \max\{(\bar{f}_i - f_i)/f_i\} \quad (22a)$$

$$\text{Err} = \max\{(1/\bar{f}_i - 1/f_i) \cdot f_i\} \quad (22b)$$

Example 1: Master Curves of Dynamic Modulus of HS Model as Input Data

In this example, the aim is to assess the efficiency of the proposed method in dealing with the master curves of DM of asphaltic materials such as hot-mix asphalt mixtures. Therefore, the master curves of DM derived from the HS rheological model (Huet 1963; Sayegh 1965) were considered in this example despite the method being intended to be applied for mathematical models of DM. Owing to the existence of exact values of LVE properties, the accuracy of the results obtained with the proposed method can be evaluated.

The complex modulus of the HS model has been given by Nguyen et al. (2022) and Olard and Di Benedetto (2003)

$$E_{\text{HS}}^*(\omega) = E_0 + \frac{E_\infty - E_0}{1 + \delta(i\omega\tau(T))^{-k} + (i\omega\tau(T))^{-h}} \quad (23)$$

where ω = angular frequency (rad/s); and δ and $0 < k < h < 1$ = model parameters determined based on dynamic modulus data. It should be noted that $\delta = \tau(T) \cdot (E_\infty - E_0)/\eta_1$, in which $\tau(T)$ is the characteristic time and η_1 is the viscositylike parameter of the first parabolic dashpot of the HS model relating the stress to strain via a fractional differential equation (Pronk 2005). The determination of η_1 was not necessary because it does not involve in the HS model equation. In Eq. (23), E_0 , E_∞ are the value of $E^*(\omega)$ when $\omega\tau \rightarrow 0$ and $\omega\tau \rightarrow \infty$, respectively. Given that $i = \cos(\pi/2) + i \cdot \sin(\pi/2) = \exp(i\pi/2)$, Eq. (23) can be rewritten

$$E_{\text{HS}}^*(\omega) = E_0 + \frac{E_\infty - E_0}{A^2 + B^2} (A + B \cdot i) \quad (24)$$

where the values of A and B are

$$A = 1 + \delta \frac{\cos(k\pi/2)}{(\omega\tau(T))^k} + \frac{\cos(h\pi/2)}{(\omega\tau(T))^h};$$

$$B = \delta \frac{\sin(k\pi/2)}{(\omega\tau(T))^k} + \frac{\sin(h\pi/2)}{(\omega\tau(T))^h} \quad (25)$$

From Eq. (24), the exact storage, loss modulus, DM, and PA of the HS model can be easily calculated: (Nguyen et al. 2022; Olard and Di Benedetto 2003)

$$E'_{HS}(\omega) = E_0 + A \frac{E_\infty - E_0}{A^2 + B^2} \quad (26a)$$

$$E''_{HS}(\omega) = B \frac{E_\infty - E_0}{A^2 + B^2} \quad (26b)$$

$$|E^*_{HS}(\omega)| = \sqrt{(E'_{HS}(\omega))^2 + (E''_{HS}(\omega))^2} \quad (27a)$$

$$\varphi_{E,HS}(\omega) = \tan^{-1} \left[\frac{E'_{HS}(\omega)}{E''_{HS}(\omega)} \right] \quad (27b)$$

The exact values of the RM and CC can also be numerically calculated using the discrete relationships presented by Nguyen et al. (2021a, 2022).

The HS master curve of DM of the unmodified asphalt mixture studied by Nguyen et al. (2021a) was reused as input data for the proposed algorithm (Fig. 1). The best-fit HS parameters are also provided in Table 1. From Eq. (27a), the derivative of $\ln |E^*_{HS}(\omega)|$ with respect to $\ln \omega$ can be derived

$$\frac{d \ln |E^*_{HS}(\omega)|}{d \ln \omega} = \frac{\omega(E_\infty - E_0)}{|E^*_{HS}(\omega)|^2} \left[E_0(t_1 - t_2) - (E_\infty - E_0) \frac{t_2}{2A} \right]$$

$$t_1 = \frac{dA}{d\omega} \frac{1}{A^2 + B^2};$$

$$t_2 = \frac{2A}{(A^2 + B^2)^2} \left(A \frac{dA}{d\omega} + B \frac{dB}{d\omega} \right) \quad (28)$$

According to the master curves of the DM, the range of frequency of interest was fixed as $\omega \in [10^{-10}, 10^8]$ and the range of the dummy variable was chosen larger than that range, namely $u \in [10^{-25}, 10^{25}]$, to properly calculate the integral in Eq. (10). The number of the subintervals to discretize the dummy variable in the base 10 logarithmic scale was 50,000. The calculated results are summarized as follows.

The PAs versus the angular frequencies are plotted in Fig. 2(a), in which a good agreement between the computed PAs and the exact ones can be observed. The relative errors between these values were smaller than 3.3×10^{-4} at all calculating points. As a result, the computed storage and loss moduli agreed well with the exact values as shown in the Cole-Cole diagram [Fig. 2(b)]. In effect, the relative errors between the computed and the exact storage moduli calculated from Eq. (26a) were lower than 1.04×10^{-4} , and the relative errors between the computed and the exact loss moduli calculated from Eq. (26b) were lower than 1.75×10^{-4} .

By this occasion, the accuracy of the analytical approximate PAs calculated with the formulation of Booiij and Thoone (1982) was also evaluated. As can be observed in Fig. 2(a), the analytical approximate PAs were not close to the exact ones calculated from Eq. (27b). The maximal error was approximately 0.6° at the inflection point of the master curve of dynamic modulus, $\omega \approx 10^{-2.36}$ rad/s, and the

Table 1. HS parameters of the unmodified mixture studied by Nguyen et al. (2021a)

Properties	Values
E_0 (MPa)	140
E_∞ (MPa)	19,550
k	0.390
h	0.500
δ	12.000
τ (s)	20.000
T_{ref} (°C)	20

relative error at that point was 2.1×10^{-2} . Nevertheless, as the frequency approached extremely low or rather high values, the relative errors between the analytically approximated PAs and the exact values were rather large, up to 15.35%. This lack of accuracy certainly results in a shortage of accuracy of the storage and loss moduli computed, as can be seen in Fig. 2(b).

The experimental values of the PAs are also presented in Fig. 2(a), in which a relatively good agreement between the calculated and tested PAs at frequencies lower than 10^3 rad/s can be observed. At that frequency range, the absolute error between these PAs was lower than 6.16° , and the relative error was almost lower than 38.53%. However, the large discrepancy of the tested PAs did not allow the tested storage and loss moduli to form a reasonable master curve in the Cole-Cole diagram in Fig. 2(b). In effect, the errors between the experimental values and the exact ones approximately ranged from $-2,000$ to $3,100$ MPa. In terms of relative errors, the experimental storage moduli differed $<15\%$ from the exact ones, whereas the experimental loss moduli may differ $>200\%$ from the exact ones at frequencies higher than 10^4 rad/s. This demonstrates that the PAs were more difficult to measure accurately than the DM.

An important step of the proposed method is to decompose the storage modulus and compliance into SEFs. By using the modified Dombi algorithm presented by Nguyen et al. (2022), the storage modulus and compliance were approximated with high accuracy by SEFs and are presented in Figs. 3(a and b). It was found that the relative errors due to the approximation process were extremely small, lower than 10^{-10} at all estimating points when 200 exponential functions were used in the sums, which can be neglected.

The RM and CC were computed using the closed-form formulas in Eq. (21) and are presented in Fig. 4. The results show that the relative errors of the obtained RM and CC were lower than 3.46×10^{-4} at all computing points. In addition, the master curve of CCs of the mixture experimentally determined by Nguyen et al. (2021a) is also shown in the figure to confirm the good agreement between the computed and experimental CCs.

For better computational efficiency of FEM analysis in the time domain, the computed RM and CC should be decomposed into SEFs. This task can be easily performed with the aid of the modified Dombi algorithm. It was shown that the relative errors between the values approximated by SEFs and the computed values were lower than 3.46×10^{-6} at every estimating point when only positive coefficients were used. Such a high level of accuracy allows one to obtain a maximal relative error of lower than 3.46×10^{-4} between the values of the SEFs and the exact values.

It is obvious that the relative error of the computed RM and CC was at the same order of magnitude as that of the computed PAs. As a result, an increase in the accuracy of the computed PAs will give rise to an increase in the accuracy of the computed LVE properties.

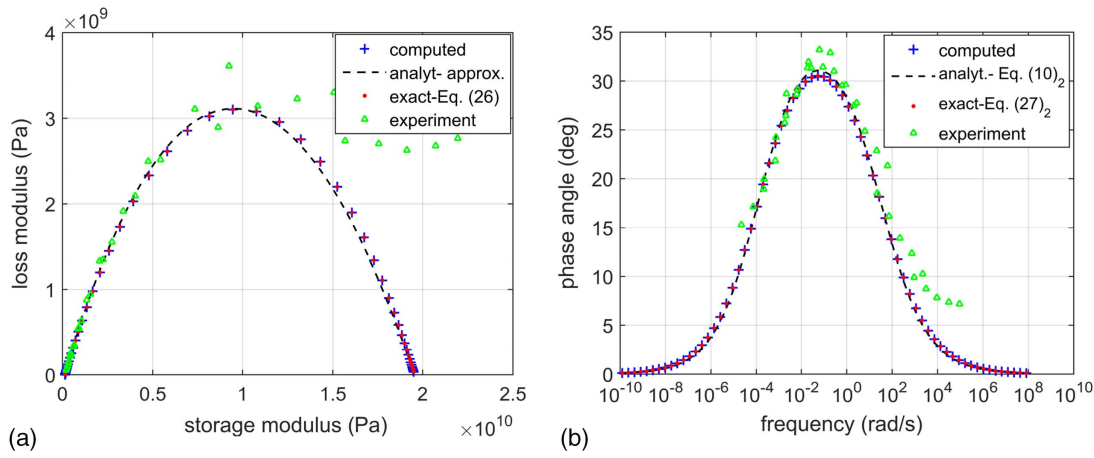


Fig. 2. (a) Phase angles obtained from HS dynamic modulus; and (b) Cole-Cole diagram obtained from HS dynamic modulus.

Example 2: MS Model

This example is devoted to demonstrating the ability of the proposed method in dealing with the MS master curve of the DM proposed by Rowe (2009) and Rowe et al. (2009)

$$\log |E_{MS}^*(\omega)| = \delta + \frac{\alpha}{\left[1 + \lambda \cdot \exp\left(\beta + \gamma \log\left(\frac{\omega}{2\pi}\right)\right)\right]^{1/\lambda}} \quad (29)$$

where δ = value of $\log |E_{MS}^*(\omega)|$ when $\omega \rightarrow 0$; $\delta + \alpha$ = value of $\log |E_{MS}^*(\omega)|$ when $\omega \rightarrow +\infty$; and λ , β , and $\gamma < 0$ = parameters controlling the shape of the function. The best-fit parameters of the MS master curve determined by Liu and Luo (2017) for an unmodified asphalt mixture at the reference temperature (20°C) were chosen as input data for the proposed algorithm, namely $\delta = 2.08$, $\alpha = 2.54$, $\lambda = 0.18$, $\beta = -0.98$, and $\gamma = -0.41$. The plot of the DM versus the angular frequency is shown in Fig. 5(a). The angular frequency was used in this study instead of the frequency to maintain the consistency of the formulation throughout the paper.

The corresponding LVE properties such as PA, storage modulus, RM, and CC can be analytically approximated as follows. Letting $v = \beta + \gamma \log(\omega/2\pi)$, the derivative of $\log |E^*(\omega)|$ with respect to $\log \omega$ can be derived from Eq. (29) in a straightforward manner

$$\frac{d \log |E_{MS}^*(\omega)|}{d \log \omega} = -\frac{\alpha \gamma e^v}{(1 + \lambda \cdot e^v)^{1+1/\lambda}} \quad (30)$$

The PA can be analytically approximated using the formula of Booiij and Thoone (1982)

$$\varphi_{E,MS}(\omega) \approx -\frac{\pi}{2} \cdot \frac{\alpha \gamma e^v}{(1 + \lambda \cdot e^v)^{1+1/\lambda}} \quad (31)$$

based on which the storage modulus and its logarithm function can be approximated by

$$\begin{aligned} E'_{MS} &\approx |E_{MS}^*(\omega)| \cdot \cos(\varphi_{E,MS}(\omega)); \\ \log(E'_{MS}) &\approx \log |E_{MS}^*(\omega)| + \log[\cos(\varphi_{E,MS}(\omega))] \end{aligned} \quad (32)$$

According to the work of Schapery and Park (1999), the RM and CC can be analytically approximated based on the master curve of the storage modulus

$$\begin{aligned} E_{MS}\left(t = \frac{1}{\omega}\right) &\approx \frac{E'_{MS}(\omega)}{\lambda'(\omega)}; \quad D_{MS}\left(t = \frac{1}{\omega}\right) \approx \frac{\sin(n\pi)}{n\pi} \cdot \frac{\lambda'(\omega)}{E'_{MS}(\omega)}, \\ \lambda'(\omega) &= \Gamma(1-n) \cos\left(n \frac{\pi}{2}\right) \end{aligned} \quad (33)$$

where Γ = gamma function; and n = derivative of $\log(E_s(\omega))$ with respect to $\log \omega$, which can be explicitly computed

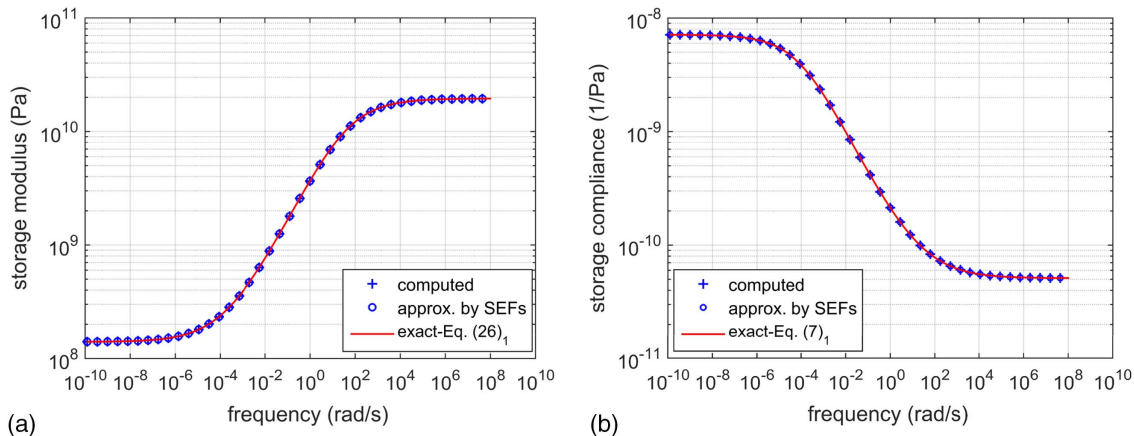


Fig. 3. (a) Approximation by SEFs of storage moduli computed from the HS dynamic modulus; and (b) approximation by SEFs of storage compliances computed from the HS dynamic modulus.

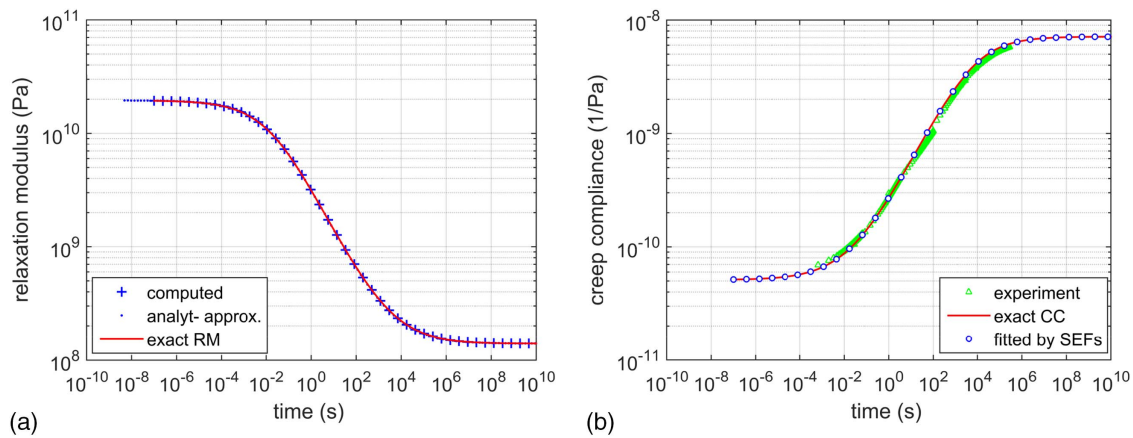


Fig. 4. (a) Computed relaxation moduli from HS dynamic modulus and their approximations by SEFs; and (b) computed CCs from HS dynamic modulus and their approximations by SEFs.

$$\frac{d \log(E'_{MS}(\omega))}{d \log \omega} \approx - \frac{\alpha \gamma e^v}{(1 + \lambda \cdot e^v)^{1+1/\lambda}} - \varphi_E^{MS} \cdot \tan \varphi_E^{MS} \cdot \frac{\gamma}{\ln 10} \cdot \frac{\lambda(e^v - 1)}{1 + \lambda e^v} \quad (34)$$

Owing to the lack of exact formulas for LVE properties corresponding to the MS master curve of DM, the analytical approximate values will be used as reference values to verify the reasonability of the results computed with the proposed method.

The computed results are summarized as follows. First, the PAs corresponding to the frequency of interest in the range $[10^{-10}, 10^{10}]$ were computed using Eq. (13). The range of the dummy variable was chosen $u \in [10^{-25}, 10^{25}]$, which was divided into 50,000 sub-intervals in the base 10 logarithmic scale. The computed and the analytical approximate PAs calculated from Eq. (31) are plotted in Fig. 5(b). It was found that the difference between these values was rather small (lower than 0.7°). However, the relative difference is important, which may exceed 2×10^{-1} at frequencies lower than 5×10^{-8} rad/s. At this stage, we cannot conclude whether the computed or the analytical approximate PA is more accurate because the exact values of the PAs do not exist. Thus, the LVE properties were calculated based on both the proposed method and the analytical approximation.

The storage modulus and compliance were computed then approximated by the SEFs. The number of exponential functions used

was 97 for the storage modulus and 74 for the storage compliance, which allowed an extremely small relative error of lower than 8×10^{-8} at all estimating points. The RM and CC were then obtained by using the closed-form formulas in Eq. (21) and are presented in Fig. 6. The analytical approximate results are also presented in the figure to verify the agreement between the results obtained with the two different approaches. It was found that the relative difference between the results was rather small, lower than 2.7×10^{-2} at all estimating frequencies. Thus, the proposed algorithm allows determining reasonable values of LVE properties based only on the master curve of the DM. Furthermore, the computed RM and CC were also decomposed with high accuracy into SEFs for better computational efficiency in FEM analysis in the time domain (Fig. 6). The error of the approximation by SEFs was very small even only positive coefficients were used, namely the relative error was lower than 3.8×10^{-5} at all estimating frequencies.

In addition, the obtained RM in the form of the SEFs was used to backward compute the DM. Because the exact value of the DM is known a priori, the accuracy of the proposed method can be verified. Because a couple of the coefficients and decay rates of the exponential functions of the RM are equivalent to the stiffness and relaxation times of one branch in the GM model, the DM can be easily interconverted from the RM. Fig. 7 plots the dynamic moduli interconverted from the master curves of the RM against the input dynamic moduli to verify the equality. All dynamic moduli

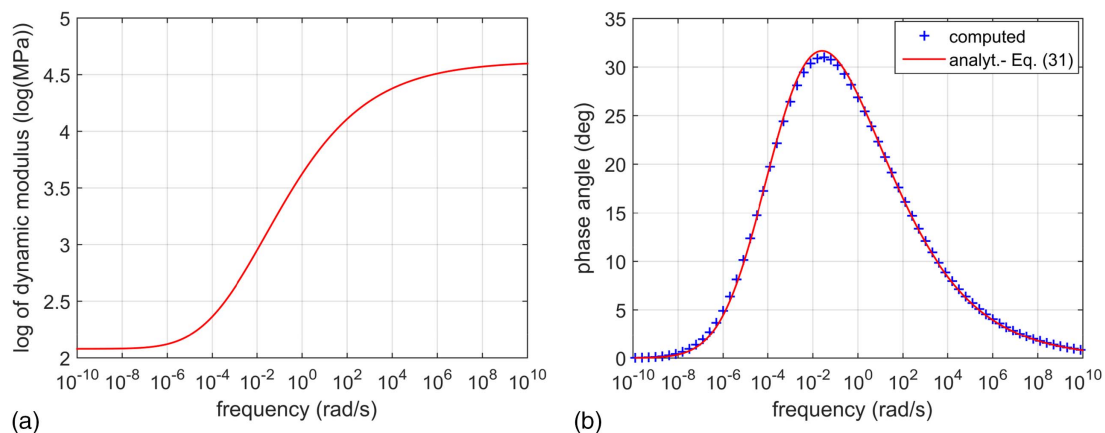


Fig. 5. (a) MS master curve of DM data (data from Liu and Luo 2017); and (b) master curve of computed PAs from the MS master curve of DM.

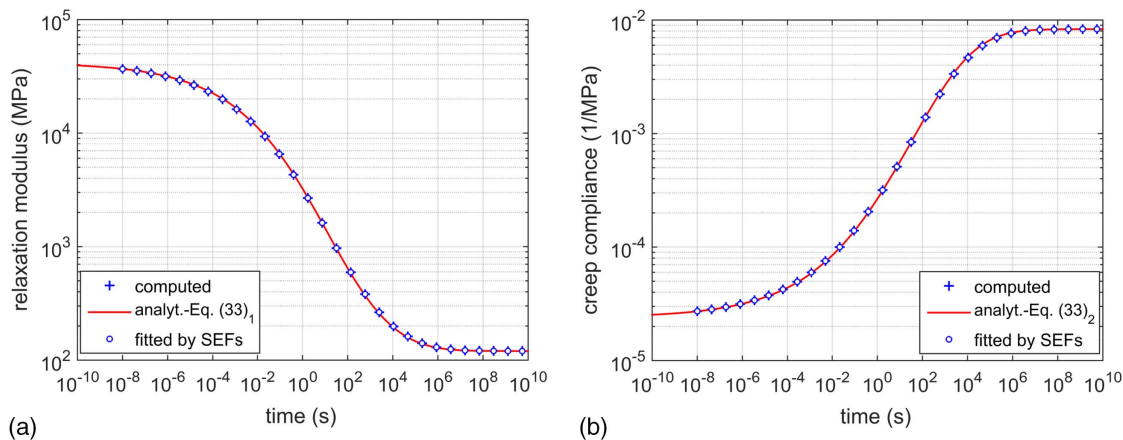


Fig. 6. (a) Relaxation moduli computed from the MS master curve of DM and their approximations by SEFs; and (b) creep compliances computed from the MS master curve of DM and their approximations by SEFs.

backward calculated from the relaxation moduli computed with the proposed method fell on the equality line, whereas those interconverted from the analytical approximate relaxation moduli slightly deviated from the equality line. The relative errors between the backward calculated and the input dynamic moduli were lower than 3.4×10^{-4} when the numerical relaxation moduli were used, and lower than 2.7×10^{-2} when the analytical approximate relaxation moduli were used. Thus, the proposed formulation achieved a much better level of accuracy than the analytical approximate formulation. The relative error arising in the proposed algorithm is so small that it can be considered neglected and covered by the error of the process of fitting experimental data by LVE models.

Conclusions

A novel method for determining with high accuracy the RM and CC of asphaltic materials in the form of SEFs based only on the master curves of DM has been successfully developed. In the formulation, the PA was numerically calculated from the DM master curve on the basis of the exact Kramers-Kronig relationships. Owing to the special form of the SEFs used for approximating with high accuracy the storage modulus and compliance, the inverse Fourier transform was analytically performed to derive the closed-form formulas of the RM and CC. Thus, the proposed formulation can be considered to fully

obey the LVE theory and physical causality principle, giving rise to an excellent level of accuracy of the obtained results. The main findings of this work can be drawn as follows:

- The relative error of the PAs calculated was very small despite a rather coarse frequency step that was used, namely lower than 1.75×10^{-4} at all computing points.
- The maximal relative error of the approximation by SEFs using the modified Dombi algorithm was extremely small, namely lower than 8×10^{-8} when using both negative and positive coefficients, which can be considered neglected.
- The inverse Fourier transform was exactly performed to calculate the RM and CC thanks to the special form of the SEFs.
- The computed RM and CC were also approximated with high accuracy by SEFs using the modified Dombi algorithm. The maximal relative error was lower than 3.8×10^{-5} when using only positive coefficients.

The level of the overall accuracy of all computed LVE properties was dominated by that of the PA. Therefore, more accurate results can be obtained by just improving the accuracy of the numerical integration in the PA calculation step. Although the proposed algorithm for determining the RM and CC in the forms of SEFs requires more calculation steps, the obtained results are much more accurate, which will contribute to better analysis and design of asphaltic pavement, and therefore a longer service life. The application of the method for DM master curves of asphalt binder can be carried out in a similar manner yet was not presented due to the page length limitation.

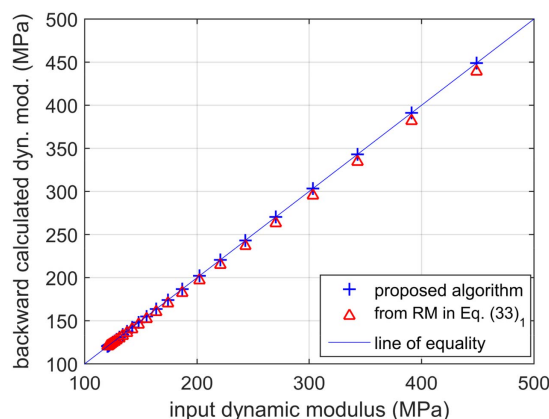


Fig. 7. Check of equality between the input dynamic moduli and those interconverted from the relaxation modulus computed.

Appendix I. Proof of Eq. (21)

By substituting Eq. (20) into Eq. (18), one has

$$E(t) = \frac{2E^*(\omega \rightarrow \infty)}{\pi} \int_0^{+\infty} \frac{\sin(\omega \cdot t)}{\omega} d\omega - \frac{2}{\pi} \int_0^{+\infty} \sum_{j=1}^m E'_j e^{b_j \omega} \frac{\sin(\omega \cdot t)}{\omega} d\omega \quad (35)$$

where the first integral in the second member is equal to $E^*(\omega \rightarrow \infty)$. The remaining integral in Eq. (35) can be calculated as follows:

$$\begin{aligned}
& \frac{2}{\pi} \int_0^{+\infty} \sum_{j=1}^m E'_j e^{b_j \omega} \frac{\sin(\omega \cdot t)}{\omega} d\omega \\
&= \frac{2}{\pi} \sum_{j=1}^m E'_j \cdot \int_0^{+\infty} L\{u[s - (-b_j)]\}(\omega) \cdot \sin(\omega \cdot t) d\omega \\
&= \frac{2}{\pi} \sum_{j=1}^m E'_j \cdot \int_0^{+\infty} u[\omega - (-b_j)] \cdot L\{\sin(s \cdot t)\}(\omega) d\omega \\
&= \frac{2}{\pi} \sum_{j=1}^m E'_j \cdot \int_0^{+\infty} \frac{t}{\omega^2 + t^2} d\omega \quad (36)
\end{aligned}$$

where L = Laplace transform; $u(s - a)$ = delay function such that $u(s - a) = 0$ if $s < a$; $u(s - a) = 1$ if $s \geq a$. Performing change of variable $v = \omega/t$ gives rise to

$$\frac{2}{\pi} \sum_{j=1}^m E'_j \cdot \int_{-b_j}^{+\infty} \frac{d\omega/t}{(\omega/t)^2 + 1} = \frac{2}{\pi} \sum_{j=1}^m E'_j \cdot \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{-b_j}{t} \right) \right) \quad (37)$$

In addition, $\tan^{-1}(-b_j/t) + \tan^{-1}(-t/b_j) = \pi/2$. Thus, Eq. (21) is obtained.

Appendix II. Modified Dombi Algorithm

Let $f(t)$ be a decaying function in the time range of interest $[0, t_n]$, which is divided by n equidistant time steps $[0, t_n] = \bigcup_{i=0}^{n-1} [t_i, t_{i+1}]$, $\Delta t = t_{i+1} - t_i$. The approximated value of $f(t)$ at time t_i is

$$\tilde{f}(t_i) = \sum_{j=1}^m a_j e^{b_j(i\Delta t)} = \sum_{j=1}^m a_j (e^{b_j\Delta t})^i \quad (38)$$

Let $q_j = e^{b_j\Delta t}$, $j = 1, \dots, m$. It can be demonstrated that if $q_1, q_2, \dots, q_m$ are the root of the m -order polynomial ($2m \leq n$)

$$q^m + s_1 q^{m-1} + s_2 q^{m-2} + \dots + s_m = 0 \quad (39)$$

then the coefficients of the m -order polynomial are the root of the following equations (Dombi 2005):

$$\begin{aligned}
& \tilde{f}_1 s_m + \tilde{f}_2 s_{m-1} + \dots + \tilde{f}_m s_1 + \tilde{f}_{m+1} = 0 \dots \\
& \tilde{f}_m s_m + \tilde{f}_{m+1} s_{m-1} + \dots + \tilde{f}_{2m-1} s_1 + \tilde{f}_{2m} = 0 \dots \\
& \tilde{f}_{n-m} s_m + \tilde{f}_{n-m+1} s_{m-1} + \dots + \tilde{f}_{n-1} s_1 + \tilde{f}_n = 0 \quad (40)
\end{aligned}$$

The decay rates of the exponential functions can be simply calculated by

$$b_j = \ln(q_j)/\Delta t \quad (41)$$

Once the decay rates have been determined, the coefficients of the exponential functions can be optimized using linear least-square techniques to best fit the input data.

Because the original method of Dombi (2005) works only with a constant time step, the method cannot be directly applied to LVE functions, which must be evaluated over several decades. Therefore, Nguyen et al. (2022) have proposed a modified version of the original Dombi algorithm. The time range was divided into several smaller intervals of time and the appropriate decay rates in each single time interval were determined. The appropriate decay rates of the input function over the entire time range of interest are just the combination of all obtained suitable decay rates.

Another remark is that the algorithm works only with decaying functions. Therefore, the following pretreatments to the input

function should be done before performing the decomposition algorithm to ensure numerical convergence:

$$\begin{aligned}
E'(\omega) &= E^*(\omega \rightarrow \infty) - E'(\omega); \\
D'(t) &= D'(\omega) - D^*(\omega \rightarrow \infty) \quad (42)
\end{aligned}$$

$$\begin{aligned}
E(t) &= E(t) - E^*(t \rightarrow 0); \\
D(t) &= D^*(t \rightarrow \infty) - D(t) \quad (43)
\end{aligned}$$

The obtained approximations by sums of exponential functions are therefore

$$\begin{aligned}
E'(\omega) &= E^*(\omega \rightarrow \infty) - \sum_{j=1}^{N_j} E'_j e^{b_j \omega}; \\
D'(t) &= \sum_{j=1}^{N_j} D'_j e^{c_j \omega} - D^*(\omega \rightarrow \infty) \quad (44)
\end{aligned}$$

$$\begin{aligned}
E(t) &= E(t \rightarrow 0) + \sum_{j=1}^{N'_j} E_j e^{b'_j t}; \\
D(t) &= D(t \rightarrow \infty) - \sum_{j=1}^{N'_j} D_j e^{c'_j t} \quad (45)
\end{aligned}$$

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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Development of a generalised creep-recovery test and a back-calculation method for determining the permanent deformation of asphalt mixtures in the time domain

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ABSTRACT

The measured strain during a creep-recovery (CR) test comprises both reversible and irreversible strains; therefore, adequate treatment is required to accurately separate them. In this paper, a generalised version of the CR test was proposed to investigate the permanent deformation behaviour of asphalt mixtures exerted by a variety of loading patterns in the time domain. The companion back-calculation algorithm was also proposed to separate with improved accuracy and repeatability the pure viscoelastic (VE) and viscoplastic (VP) strains. First, the creep compliance (CC) was back-calculated based only on the recovery data by using an approximate analytical creep compliance function of the Huet-Sayegh model as the solution function in the optimisation process. Then, the pure VE and VP strains were determined. The results show that the proposed method attains a good level of reliability although relatively short durations of the recovery period have been used.

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1. Introduction

Since asphalt mixtures have viscous properties, the total recovery of the resilient strain takes a long time to finish. As a result, current rutting characterisation procedures such as the Hamburg Wheel Tracking (HWT) test (AASHTO T324, 2004; EN 12697-22, 2007) have difficulties in determining accurately the net value of the permanent strain, or the viscoplastic (VP) strain. The deformation measured after each load cycle comprises not only the irreversible deformation but also a part of the viscoelastic (VE) deformation that has not yet recovered. Moreover, the stress and strain fields in the HWT test are not homogeneous; therefore, the HWT test is often considered an empirical one and is primarily used to control the mix design (Setra-LCPC, 1994). In a recent attempt, Nguyen (2017) reported that the use of HWT test data to predict the rutting distress of flexible pavement resulted in an overestimation, especially in high-speed traffic areas. This may be due to the fact that the model parameters were not correctly determined because of the permanent deformation (PD) being measured inaccurately.

In contrast, creep/creep-recovery tests are homogeneous tests and can be performed in both static and cyclic loading; thus, the test results can be easily used to determine the prediction model parameters. The application of the creep test on asphalt mixtures is not recent and can be dated back to the pioneer work of Van der Loo (1978). It was recommended that the results of creep tests were used

for ranking the rutting potential of asphalt mixtures rather than for predicting the onsite PD of pavements (On Tam et al., 2000). In fact, the measured deformation during a creep test does comprise the viscoelastic strain because no recovery phase is allowed. In order to overcome this drawback, Sun and Zhu (2013) performed dynamic modulus (DM) tests to characterise the VE properties of the material before conducting creep tests. As a result, the VE creep strain can be predicted and therefore the VP strain evolving during the test can be determined. Such a two-stage process has some disadvantages regarding inter-sample variations and experiment time. In fact, the master curve of the complex modulus must be constructed over a wide range of temperature and frequency so as to accurately compute the master curve of the creep compliance (CC), while only a short portion of the CC master curve at a single temperature is needed.

Meanwhile, a creep-recovery (CR) test consists of two phases: a loading one followed by a resting one, which allows separating the reversible and irreversible strains from the total strain measured (Drescher et al., 1993). Many testing procedures and calculation algorithms have been proposed to determine the VP properties and/or VE properties based on CR test data (Daoudi et al., 2020; Liu & You, 2009; Nguyen et al., 2021). The accuracy of the separated permanent strain depends on the calculation algorithm proposed and the duration of the resting period used.

On Tam et al. (2000) performed a 60-minute loading period followed by a 10-minute resting period and considered the deformation at the end of the test as the PD. As will be demonstrated later, such a short resting time could not allow a full recovery of the VE strain, which certainly results in a lack of accuracy achieved. In a later work, Levenberg (2006) used the data in the recovery phase to determine the VE response with the aid of an undisclosed nonlinear optimisation algorithm, after which the permanent strain could be calculated based on the data in the loading phase. Later, Abu Al-Rub et al. (2009) continued to mine the data in the recovery phase to characterise the VE properties of the material before calculating the VP strain. In addition, the simplified nonlinear VE theory of Schapery (1969) was also integrated into the formulation to consider the effects of high-stress levels. The Prony series of the CC function was chosen as the solution function to the problem, in which the values of the retardation times were chosen a priori so as to turn the nonlinear least-square optimisation problem into an easier linear one. The method was then widely adopted by many researchers, e.g. Bai et al. (2014), and Chen et al. (2017).

However, the direct fitting of experimental data by Prony series-based functions without pre-smooth treatment usually causes artificial waviness to the predicted results and thus a loss of accuracy, which will be illustrated in the subsequent Sections. In an attempt to overcome this shortcoming, Darabi et al. (2019) proposed a two-stage algorithm, which combines the advantages of the two methods of Sun and Zhu (2013) and Abu Al-Rub et al. (2009). First, the VE properties of the material were characterised using the DM test, after which the Prony series associated with the creep compliance was determined. Subsequently, a series of creep-recovery test data were used to identify the nonlinear VE parameters before determining the VP strain. The method was able to avoid spurious oscillations of the creep compliances determined but still had the same disadvantages as the method of Sun and Zhu (2013).

All the methods mentioned above used the single-stress level creep-recovery (SSCR) test to characterise the pure VP behaviour of mixtures except (Bai et al., 2014) wherein a multi-stress level creep-recovery (MSCR) test was used instead. However, in that study, the behaviour of the material in each CR cycle was assumed equivalent to that occurring in an SSCR test; the influence of past creep-recovery cycles on the actual one was not considered yet. Therefore, the objective of this work is to propose a generalised version of the SSCR test and a companion back-calculation algorithm to determine the pure VP strain of asphalt mixtures undergoing any complex loading history. The sought formulation must involve only the results of the generalised creep-recovery (GCR) test at a single testing temperature and overcome all the above-mentioned shortcomings. In addition, the results obtained with the proposed method should achieve a good level of accuracy and repeatability.

We shall restrict ourselves to the *small VE strain* assumption for the Boltzmann superposition principle to be applicable. As a result, the back-calculated CCs, an intermediate result, can be compared

with the true values to them calculated by interconverting the data of the DM test – a standardised and widely accepted test procedure.

2. Materials and methods

2.1. Materials and samples preparation

This study re-examined the behaviour of the highly modified asphalt mixture studied in (Nguyen et al., 2021). The mixture was intentionally designed to possess a significant resistance to rutting for being used in heavy-duty pavements. It is a mixture of well-graded aggregate of 12.5 mm of nominal maximum aggregate size and a highly modified bitumen of performance grade PG 94. The aggregate gradation curve and the fundamental properties of the mixture have been reported in (Nguyen et al., 2021) and are not presented here.

In order to prepare the samples for VE and VP characterisation, the mixtures were first compacted by a roller sector compactor to obtain a plate with dimensions of $0.32 \times 0.26 \times 0.08 \text{ m}^3$. The plate was then sawed along its width to obtain three prismatic samples of dimensions $0.07 \times 0.07 \times 0.15 \text{ m}^3$, as shown in Figure 1. The sample coded #1 was dedicated to the determination of complex moduli and CCs at intermediate and high temperatures, which were already carried out and reported in (Nguyen et al., 2021). The remaining two samples (coded as #2 and #3) were dedicated to the characterisation of PD carried out in this study.

It was shown in (Nguyen et al., 2021) that the mixture fully obeyed the time-temperature superposition principle (TTSP), and its complex modulus could be well predicted by the Huet-Sayegh (HS) model (Olard & Di Benedetto, 2003; Sayegh, 1965). The HS parameters of the mixture are reported in Table 1, and the shift factors can be predicted by a function of temperature T (°C): $a_T = \exp(a.T^2 + b.T + c)$ with $a = 0.0012$, $b = -0.3539$, and $c = 6.6854$.

2.2. Test set-up

The examined specimen was placed between the upper and lower load platens, based on which a controlled compressive force was applied to the specimen (see Figure 2). It should be noted that the lower load platen was the driving one while the upper load platen was the fixed one. Two displacement transducers were installed along the height of the specimen to measure the relative displacement

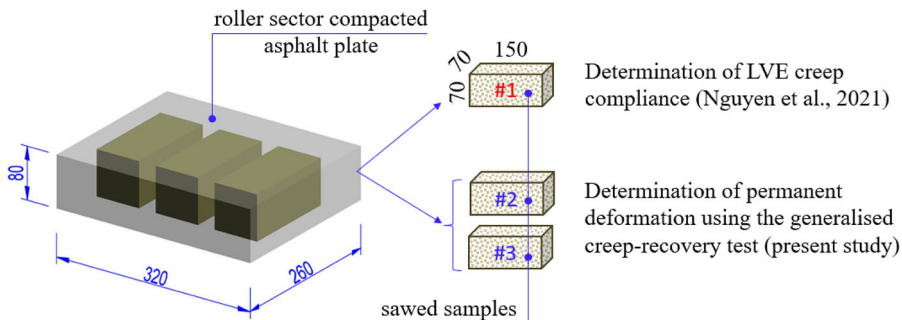


Figure 1. Samples preparation for the CR test studied in Nguyen et al. (2021) and for the GCR tests performed in the present research.

Table 1. HS parameters of the modified mixture determined in Nguyen et al. (2021).

E_0 (MPa)	E_∞ (MPa)	k	h	δ	τ (s)	T_{ref} (°C)
139	45,000	0.250	0.500	4.938	16.302	20

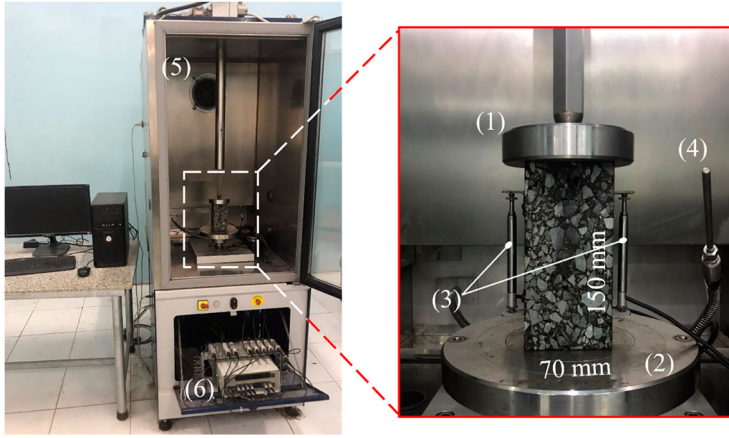


Figure 2. Test set-up for the GCR test: (1) Upper load platen, (2) Lower load platen, (3) Displacement transducer, (4) Control temperature sensor, (5) Thermal chamber, (6) Control and Data Acquisition System.

between the fixing points initially distanced by 85 mm. The specimen and measuring devices were all placed in a thermal chamber to maintain a constant temperature during the test.

2.3. Generalised creep-recovery test procedure and back-calculation method

Let us assume that the sample is under zero-strain and zero-stress conditions at the beginning of the test, i.e. $\sigma(t) = 0$ and $\varepsilon(t) = 0$ for $t \leq t_0$. The sample is then applied by a stress history in the form of a step function such that $\sigma(t) = \sigma_i$ for $t_{i-1} < t \leq t_i$, with $t_i = t_0 + \sum_{j=1}^i t_{L,j}$, $i \in \{1, ..n\}$, and $\sigma(t) = 0$ for $t > t_n$ as shown in Figure 3(a). For simplicity, we shall hereafter denote the time interval from t_0 to t_n as the *loading period* and the time interval from t_n to $t_n + t_{uL}$ as the *recovery period*. By far, the number of applied loading steps n and the prescribed stress level is allowed to be freely chosen parameters.

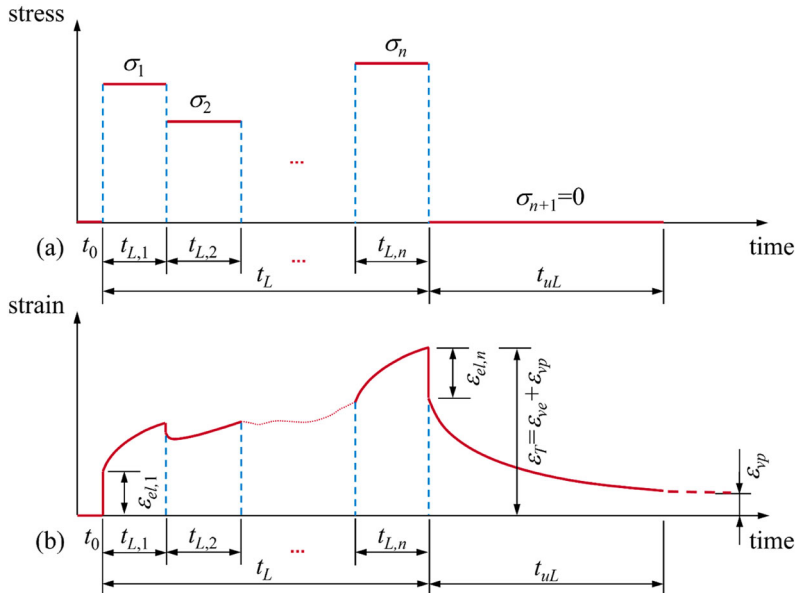


Figure 3. Description of the GCR test, (a) applied stress history, (b) response strain.

The general form of the response strain under such a stress history is sketched in Figure 3(b). For asphalt mixtures, VP strain develops even at a low temperature and stress level (Nguyen et al., 2021); therefore, the measured strain is the summation of VE and VP strains:

$$\varepsilon_T(t) = \varepsilon_{ve}(t) + \varepsilon_{vp}(t) \quad (1)$$

where ε_T , ε_{ve} and ε_{vp} respectively denote the total, VE, and VP strain.

It should be noted that damage may occur under tensile stress or very high compressive stress conditions. Thus, in order to study the net PD behaviour of asphalt mixtures, the applied stress in this work was in compression and its value was limited to be small enough so that damage can be neglected.

Let us define the increment of the stress input at a time t_i as $\Delta\sigma_i = \sigma_{i+1} - \sigma_i$ with $i \in \{0, \dots, n-1\}$ and assume that the VE strain is small enough so that the Boltzmann superposition principle can be applied. The formula for calculating the VE strain can be expressed as:

$$\varepsilon_{ve}(t) = \sum_{i=0}^{m, t_m < t \leq t_{m+1}} D(t - t_i) \Delta\sigma_i, \text{ for } t_0 < t \leq t_n \quad (2)$$

$$\varepsilon_{ve}(t) = \sum_{i=0}^{n-1} D(t - t_i) \Delta\sigma_i - D(t - t_n) \cdot \sigma_n, \text{ for } t > t_n \quad (3)$$

In Equations (2) and (3), the VE strain is not right continuous but still left continuous at $t_i, i \in \{0, \dots, n\}$ due to the specified form of the input stress history, which can be expressed as:

$$\varepsilon_{ve}(t_i^-) = \varepsilon_{ve}(t_i); \varepsilon_{ve}(t_i^+) \neq \varepsilon_{ve}(t_i) \text{ for } i \in \{0, \dots, n\} \quad (4)$$

It should be noted that the applied stress in current rutting characterisation procedures, e.g. HWT test (AASHTO T324, 2004; EN 12697-22, 2007), may exceed the linear viscoelastic (LVE) domain limitation. As a result, the Boltzmann superposition principle cannot be applied. In this case, nonlinear VE theories such as the one proposed by Schapery (1969) must be used to formulate the stress-VE strain relationship. However, we still limit ourselves to the LVE domain in this study so as to simplify the problem. The extension of the method to deal with nonlinear viscoelasticity will be considered in future works.

Since the stress in the resting period is equal to zero, one can assume that the VP strain does not evolve during this period. This means that ε_{vp} does not change from time t_n onwards:

$$\varepsilon_{vp}(t) = \varepsilon_{vp}(t_n), \text{ for } t \geq t_n, \text{ noted } \varepsilon_{vp} \quad (5)$$

As a result, the VP strain is continuous at t_n – namely,

$$\varepsilon_{vp}(t_n^-) = \varepsilon_{vp}(t_n^+) = \varepsilon_{vp} \quad (6)$$

The total strain we measure from the end of the loading period to the end of the recovery period is:

$$\varepsilon_T(t) = \varepsilon_{ve}(t) + \varepsilon_{vp}, \text{ for } t \geq t_n \quad (7)$$

from which the measured strain at time t_n can be expressed by:

$$\varepsilon_T(t_n) = \varepsilon_{ve}(t_n) + \varepsilon_{vp} \quad (8)$$

Subtracting respectively two members of Equation (8) by those of Equation (7) gives rise to:

$$\varepsilon_T(t_n) - \varepsilon_T(t) = \varepsilon_{ve}(t_n) - \varepsilon_{ve}(t), \text{ for } t \geq t_n \quad (9)$$

It can be seen in Equation (9) that the VP strain has been eliminated and just the VE and the measured strains remain in the equation. Substituting Equations (2) and (3) into Equation (9) results in an

equation comprising only the CC function, the prescribed stress, and the measured strain:

$$\frac{\varepsilon_T(t_n) - \varepsilon_T(t)}{\sigma_n} = \sum_{i=0}^{n-1} D(t_n - t_i) \frac{\Delta\sigma_i}{\sigma_n} - \sum_{i=0}^{n-1} D(t - t_i) \frac{\Delta\sigma_i}{\sigma_n} + D(t - t_n), \text{ for } t > t_n \quad (10)$$

Both sides of Equation (10) have the dimension of compliance and will hereafter be called ‘recovery compliance’. It is worth mentioning that the value of the left-hand side of Equation (10) can be easily computed from the GCR test data while the stress levels and the number of loading intervals are user-defined parameters. Thus, the *unknown* of Equation (10) is just the *CC function* $D(t)$. The functional equation (10) can be solved using optimisation techniques provided that the form of the CC function is known a priori.

Once the functional Equation (10) has been resolved, the CC compliance function is known and the VE strain at any time can be computed using Equation (2) or Equation (3). Consequently, the *pure VP strain* at time t can be calculated by subtracting the total strain by the VE strain:

$$\varepsilon_{vp}(t) = \varepsilon_T(t) - \varepsilon_{ve}(t) \quad (11)$$

Nguyen et al. (2021) demonstrated that the three-parameter power creep function and the analytical approximate CC (AACC) function derived from HS and 2S2P1D model could be used for the resolution of Equation (10). However, the solution obtained with the three-parameter power creep function did not match very well with the real behaviour of asphalt mixtures at high temperatures due to the fact that too few parameters were used in the formula. On the contrary, owing to the excellence in fitting LVE behaviour of asphalt mixtures of HS and 2S2P1D model, the AACC function allowed obtaining good solutions even at high temperatures. Therefore, only the AACC function derived from HS or its generalised version, 2S2P1D model, was considered in this study. The complex modulus formulas of HS and 2S2P1D models are given in (Di Benedetto et al., 2004; Olard & Di Benedetto, 2003):

$$E_{HS}^*(\omega) = E_0 + \frac{E_\infty - E_0}{1 + \delta(i\omega\tau(T))^{-k} + (i\omega\tau(T))^{-h}} \quad (12)$$

$$E_{2S2P1D}^*(\omega) = E_0 + \frac{E_0 - E_\infty}{1 + \delta(i\omega\tau(T))^{-k} + (i\omega\tau(T))^{-h} + (i\omega\beta\tau(T))^{-1}} \quad (13)$$

where ω (rad/s) is the angular frequency, δ , $0 < k < h < 1$ are model parameters and E_0, E_∞ are the value of $E^*(\omega)$ when $\omega\tau \rightarrow 0$ and $\omega\tau \rightarrow \infty$ respectively, and β is also a material parameter. The CC function can be analytically approximated by using the method of Schapery and Park (1999):

$$D\left(t = \frac{1}{\omega}\right) = \frac{\sin(n\pi)}{n\pi} \cdot \frac{\lambda'(\omega)}{E'(\omega)}; \lambda'(\omega) = \Gamma(1 - n) \cos\left(n\frac{\pi}{2}\right) \quad (14)$$

where $E'(\omega)$ is the storage modulus, Γ is Gamma function, and n is the slope of the curve $\log E'(\omega)$ versus $\log \omega$ which can be explicitly computed as presented in (Nguyen et al., 2022). Since only a portion of the AACC function is needed to reproduce the CC at a single testing temperature, a parameter ξ was used to extract the suitable portion of the AACC function:

$$D\left(\frac{t}{\xi}\right) = \frac{\sin(n\pi)}{n\pi} \cdot \frac{\lambda'(\omega, \xi)}{E'(\omega, \xi)}; \lambda'(\omega, \xi) = \Gamma(1 - n) \cos\left(n\frac{\pi}{2}\right) \quad (15)$$

The function $D(t/\xi)$ in Equation (15) was used in the fitting process, in which all parameters including the parameter ξ are arbitrary initial values chosen such that the curve-fitting algorithm can converge to a local optimum. In this work, the HS parameters and the shift factor at 50°C obtained from DM tests in the previous study (Nguyen et al., 2021) were used as the initial point for the optimisation process. It is worth noting that the initial values do not affect the optimum solution yet improve the convergence to the right local optimum solution. The proposed formulation

for determining the *pure permanent deformation* that evolves during a *GCR test* is summarised in Algorithm 1 to facilitate the reading of this paper and to highlight the novelties of the present approach.

Algorithm 1. Algorithm for determining the pure VP strain based on the GCR test data

Input data: Time histories of stress, measured strain, number of the loading period n

- Calculate the experimental recovery compliance using Equation (10)
- Calculate the discrete stress increment $\Delta\sigma_i$, $i \in \{0, n - 1\}$

Step 1. Back-calculating the CC:

- Choose a set of initial values for the AACC function
- Use curve fitting techniques to best fit Equation (10). The AACC function with the best-fit parameters is the solution to Equation (10) using the least square means.

Step 2. Predicting the pure VE strain:

Calculate the VE strain using the best-fit parameters of the AACC function determined in Step 1:

$$\varepsilon_{ve}(t) = \sum_{i=0}^{m, t_m < t \leq t_{m+1}} D(t - t_i) \Delta\sigma_i, \text{ for } t_0 < t \leq t_n$$

$$\varepsilon_{ve}(t) = \sum_{i=0}^{n-1} D(t - t_i) \Delta\sigma_i - D(t - t_n) \cdot \sigma_n, \text{ for } t > t_n$$

Step 3. Computing the pure VP strain:

Calculate the difference between the measured strain and the predicted VE strain:

$$\varepsilon_{vp}(t) = \varepsilon_T(t) - \varepsilon_{ve}(t)$$

3. Experimental results

This Section presents the results obtained with various GCR tests, which were performed with different chosen parameters such as the number of loading intervals and the stress levels so as to demonstrate the efficiency of the proposed method in *characterising simultaneously* the VE and VP strain at a high temperature chosen equal to 50°C. The applied stress was carefully taken such that there still was a linear relationship between the stress and the VE strain. It should be noted that it is more difficult to keep the loading in the LVE domain in a creep or CR test than in a DM test because of the longer loading time. This may be the reason why there is not much research on the linearity of the VE behaviour of asphalt mixtures in creep or CR tests. For instance, Kim et al. (2008) proposed to limit the total strain during creep tests at 10°C to be lower than 450 $\mu\text{m}/\text{m}$ while Nguyen et al. (2021) chose a smaller limit of the creep strain (150 $\mu\text{m}/\text{m}$) so as to investigate the LVE behaviour of asphalt mixtures at intermediate and high temperatures. Upon checking the linearity of the stress-VE strain relationship, the stress levels were chosen lower than 35.71 kPa or such that the resultant VE strain was lower than 170 $\mu\text{m}/\text{m}$.

Three test derivatives were performed including a single-stress level creep-recovery (SSCR) test, an increasing multi-stress level creep-recovery (MSCR) test, and a decreasing one. The reason for choosing 50°C is that the time for the total recovery of the sample after releasing all the applied stresses is not too long so that the test can finish within a working day and that the properties of the sample do not change much due to the cumulative permanent strain.

The SSCR test was performed on sample #2 with four different durations of the loading period to verify the accuracy and repeatability of the obtained results. In order to verify the linearity of the stress-VE strain relationship, the effect of the stress level on the VE behaviour of the mixture was also examined. Meanwhile, sample #3 was dedicated to the increasing and decreasing MSCR tests, which were designed to analyse the pure PD of the mixture when exerted by more complicated stress histories.

It should be mentioned that the sample was left to recover its resilient deformation for at least 20 h and was conditioned in the thermal chamber for 4 h to maintain a homogenous temperature field before starting the new test procedure. Moreover, all samples had been subjected to three SSCR tests with a 600-s loading period before the official tests were performed to eliminate as much as possible

the densification caused by post-compaction. All the results were obtained based on the proposed algorithm with the aid of MATLAB R2015 software.

3.1. Single-stress level creep-recovery test (SSCR)

The results reported here were obtained with only one loading interval. Four successive replicates of the SSCR test were performed with the same compressive stress level of 35.71 kPa and different durations of the loading period (loading times): 75, 150, 300, and 600 s. After the loading time had been reached, the sample was unloaded and left to completely recover the reversible strain so as to measure the ‘true permanent strain’ developed in the sample. As a result, the resting time varied from one test to another. In order for the back-calculated results to be comparable to each other, the resting time of all tests should be the same in a relative manner. Therefore, only the data acquired over a period equal to 11 times the loading time (i.e. the resting time is 10 times the loading time) were used for back-calculation. As will be presented in the following paragraphs, such a resting time is far large enough to obtain accurate results.

Figure 4(a) shows the good fitting of the recovery compliances measured by the function described in Equation (10) in case $t_L = 300$ s. With the best-fit parameters, the corresponding CCs were computed and are shown in Figure 4(b). It can be observed that the CCs back-calculated from the data obtained with sample #2 are in full accord with those interconverted from the complex modulus data already determined with sample #1 in Nguyen et al. (2021) even at the high temperature. In fact, the relative differences between these CC curves are lower than 5.36%. Conversely, the CCs computed from creep test data (i.e. the ratio of the measured strain to the applied stress) are much higher than the CCs as can be seen in Figure 4(b), which demonstrates the existence of irreversible strain as discussed in Nguyen et al. (2021).

Once the CC of the mixture at the testing temperature had been determined, the VE strain caused by the input stress history was calculated and is shown in Figure 5(a). It is obvious that the VE strain is smaller than the measured strain, and the difference between these strains is just the VP strain shown in Figure 5(b). The VP strain steadily increased to reach its highest value at $85.5 \mu\text{m}/\text{m}$ at the end of the loading period and almost remained unchanged in the recovery period as expected. The maximal value of the measured strain was $226.13 \mu\text{m}/\text{m}$; therefore, the maximal VE and VP strains respectively account for 62.19 and 37.81% of the maximal total strain.

Figures 6(a,b) respectively display the CCs obtained with the present algorithm and the one proposed in Abu Al-Rub et al. (2009). It was shown that the relative differences between the CCs back-calculated with the present algorithm and the CCs interconverted from complex modulus data were lower than 7.47%, and the relative differences between any curve of the back-calculated CCs and

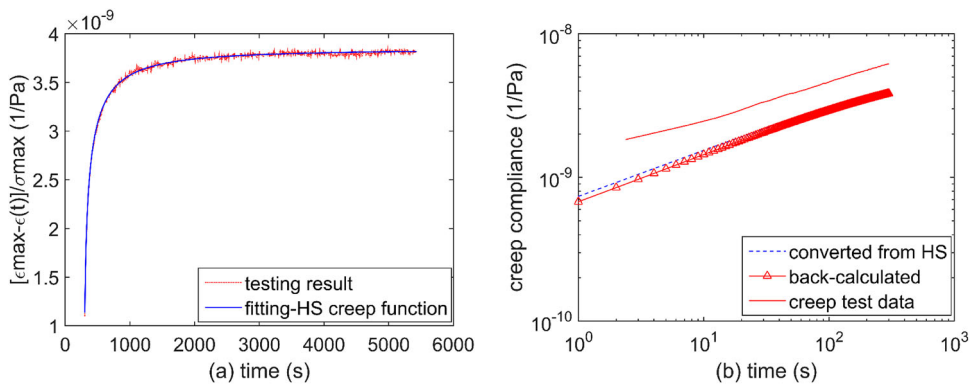


Figure 4. Results of the SSCR test at 50°C and $t_L = 300$ s, (a) Fitting results of the recovery compliance, (b) Back-calculated CC.

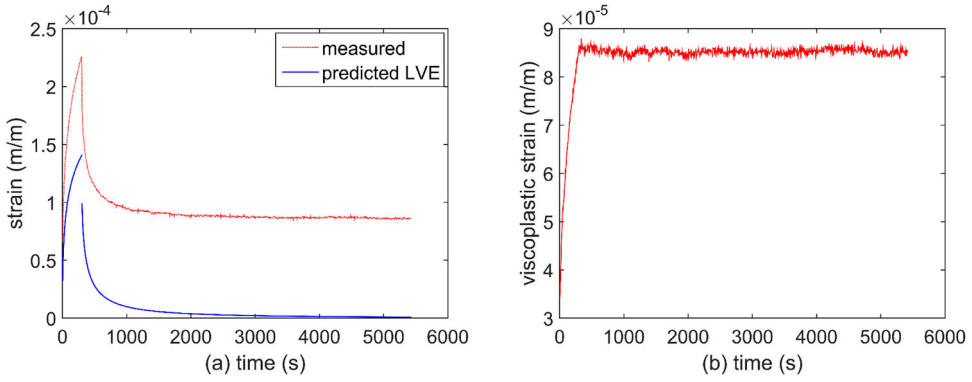


Figure 5. Results of the SSCR test at 50°C and $t_L = 300$ s, (a) Predicted VE and measured strain, (b) Computed VP strain with time.

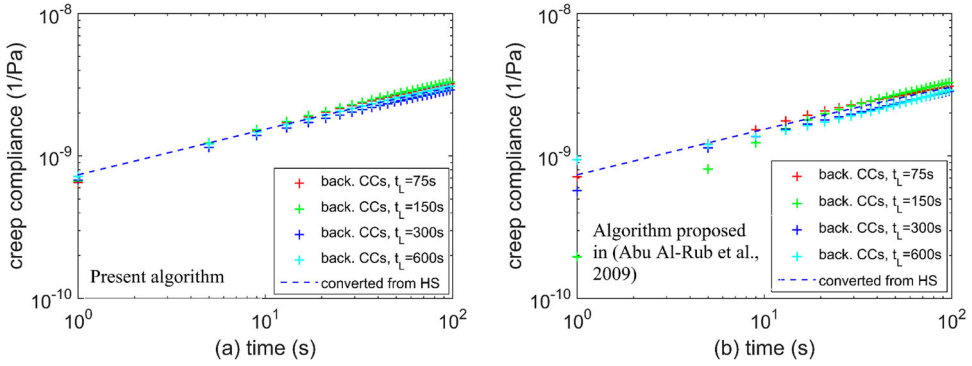


Figure 6. Back-calculated CCs from all test replicates at 50°C, (a) using the AACC function, (b) using the CC function in the form of a Prony series.

the average curve were lower than 6.68%. This again verifies the good level of reliability of the back-calculated CCs found in (Nguyen et al., 2021). In contrast, the algorithm of Abu Al-Rub et al. (2009) led to a lack of accuracy and repeatability of the obtained CCs, especially at small loading times (< 10 s). In fact, the overall relative errors (calculated with Equation (16) wherein D_i and $D_{true,i}$ is the obtained CC and that interconverted from DM data at time t_i) of the CCs reached 68%, while the relative discrepancy (calculated with Equation (17) wherein $\bar{D}_i$ is the average value of the obtained CCs at time t_i) between the computed CCs exceeded 59.9%, which is far higher than those obtained with the present algorithm. As a result, Abu Al-Rub et al. (2009)'s algorithm gave less accurate values of VP strains than the present algorithm.

$$RE = \left[\frac{\sum_i (1/D_i - 1/D_{true,i})^2}{\sum_i (1/D_{true,i})^2} \right]^{\frac{1}{2}} \quad (16)$$

$$RD = \left[\frac{\sum_i (1/D_i - 1/\bar{D}_i)^2}{\sum_i (1/\bar{D}_i)^2} \right]^{\frac{1}{2}} \quad (17)$$

Figures 7(a,b) show the VP strains obtained with both the algorithms proposed in this study and in (Abu Al-Rub et al., 2009). For comparison purposes, the exact values are also plotted in the figure. It could be demonstrated that the relative errors and the standard deviations of the VP strains obtained with the present algorithm were small, lower than 3.49% and $2.84 \mu\text{m/m}$ respectively (see Tables 2

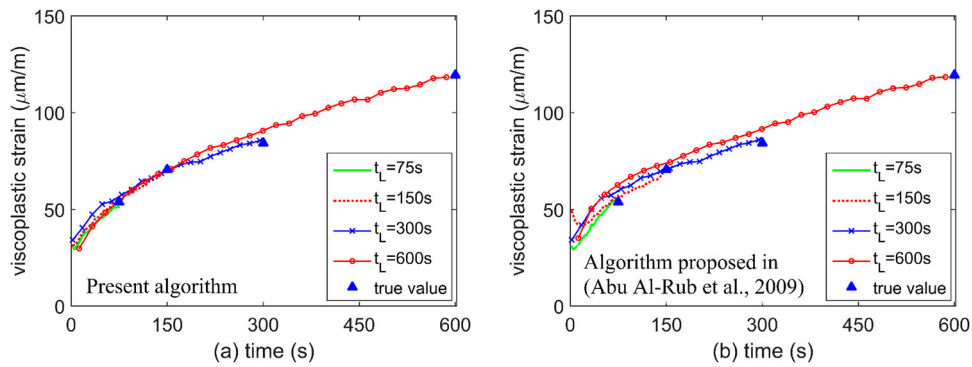


Figure 7. Pure VP strains obtained from all SSCR test replicates at 50°C, (a) using the AACC function, (b) using the CC function in the form of a Prony series.

Table 2. Assessment of the accuracy of the computed VP strains.

Loading time (s)	VP strain at time t_n		
	calculated ($\mu\text{m/m}$)	measured ($\mu\text{m/m}$)	Error (%)
Present algorithm			
75	51.94	53.82	3.49
150	70.15	70.49	0.48
300	85.72	84.29	1.69
600	119.82	119.49	0.28
Algorithm proposed in Abu Al-Rub et al. (2009)			
75	55.73	53.82	3.55
150	68.60	70.49	2.68
300	86.16	84.29	2.22
600	119.87	119.49	0.32

Table 3. Assessment of the repeatability of the computed VP strains.

At time (s)	Computed VP strain (μm/m)					
	Test replicate number				Mean	SD
	#1	#2	#3	#4		
Present algorithm						
10	31.85	33.37	37.33	28.48	32.76	2.84
75	51.94	55.79	57.03	54.43	54.80	1.69
150	–	70.15	70.22	70.32	70.23	0.06
300	–	–	85.72	91.12	88.42	2.20
600	–	–	–	119.82	–	–
Algorithm proposed in Abu Al-Rub et al. (2009)						
10	30.70	43.11	38.26	33.73	36.45	4.20
75	55.73	57.36	59.91	54.00	56.75	1.95
150	–	68.60	70.89	74.13	71.21	1.97
300	–	–	86.16	91.98	89.07	2.38
600	–	–	–	119.87	–	–

and 3). When the algorithm of Abu Al-Rub et al. (2009) was used, the maximal relative errors and the standard deviations were respectively 3.55% and 4.20 $\mu\text{m/m}$. Therefore, the proposed algorithm achieved a good level of reliability and performed better than the algorithm in (Abu Al-Rub et al., 2009). In subsequent paragraphs, only the results obtained with the present formulation will be shown for the sake of brevity.

3.1.1. Effect of resting time on the accuracy of VP strain

The effect of resting time on the accuracy of the obtained results was investigated to assess the efficiency of the proposed algorithm in terms of the testing time needed to accurately determine the *pure VP strain*. First, the data of SSCR tests in the unloading period were carefully eliminated to create the data set of SSCR tests with various unloading times ranging from 1 to 10 times the loading time, as illustrated in Figure 8. Then, the VP strains were back-calculated based on the set of the truncated data. Additionally, the 'directly measured values' of the VP strain at the end of the loading period were also determined by measuring the strain at the end of the unloading period. Consequently, the relative errors of the computed and measured VP strains at time $t = t_L$ could be calculated.

It can be observed in Figure 9 that the error of the back-calculated VP strains drastically decreased with resting time increasing up to roughly 5 times the loading time. For an unloading time greater than that value, not much extra accuracy was gained for the back-calculated VP strain. Conversely, the error of the direct measurement was still in a steady decline even though the unloading time had reached 10 times the loading time. Therefore, an unloading time equal to 5 times the loading time is sufficient for the back-calculation to achieve a good level of accuracy. In fact, all the VP strains that were determined based on the SSCR test data obtained at 50°C and with $t_{uL} = 5t_n$ differ $< 1.44\%$ from the true values except for the test with $t_L = 75s$. In a test with such a short loading time, the input stress and the response strain may fluctuate considerably due to inertial effects caused by a sudden application or removal of the stress, resulting in lower accuracy of the back-calculated results. Despite that, an accuracy level of less than 4.03% was also attained.

In comparison with the direct measurement, the proposed algorithm gave better results – namely, the error of the measured VP strain in the case $t_{uL} = t_L$ was higher than 21.6%, whereas the back-calculated VP strain determined from the data with the same resting time just differed $< 5.3\%$ from the exact value. This can be explained by the fact that the resting time taken was not long enough,

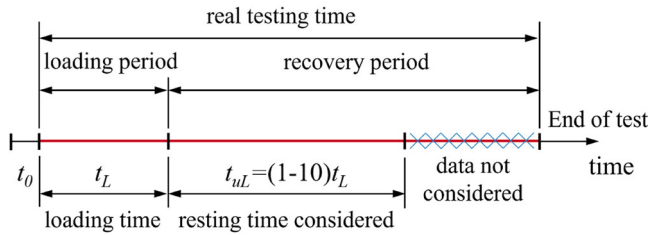


Figure 8. Illustration of constructing the truncated data used for the back-calculation algorithm.

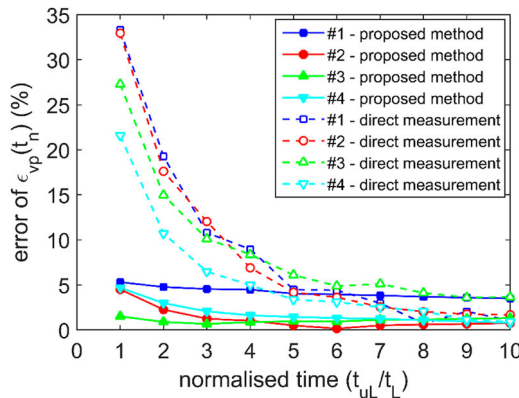


Figure 9. Effect of the resting time on the accuracy of VP strains obtained from all SSCR test replicates at 50°C.

and therefore the measured strain at the end of the resting time comprised a large amount of VE strain that has not yet recovered, resulting in a large error of the measured VP strain. According to the results obtained at 50°C, in order to achieve an accuracy level of approximately 3% for the direct measurement, the resting time must be as large as 7 times the loading time. Meanwhile, the resting time needed for the proposed algorithm to achieve the same level of accuracy is much shorter, just 2–3 times the loading time. Thus, the resting time of an SSCR test at 50°C should be chosen 2–5 times the loading time. Such values of the resting time allowed attaining a good level of accuracy while maintaining a reasonable testing time.

3.1.2. Check of linearity and effect of stress level on the VP strain

Two more SSCR tests were performed on sample #2 with stress levels equal to one-third and two-thirds of 35.71 kPa respectively. The predicted VE strains due to all investigating stress levels were computed and are presented in Figure 10. In order to check the linearity, one-third of the response VE strain caused by the stress level 35.71 kPa and one-half of the response VE strain caused by the stress level 23.81 kPa were also computed and are all shown in the figure. It can be observed that all the scaled VE strains (i.e. by a factor of 1, 1/2, and 1/3) agree well with each other – namely, the relative differences between any scaled VE strain and the average VE strain are lower than 3.95%. Thus, the relationship between the stress and the VE strain at 50°C can be considered linear for a stress level of up to 35.71 kPa or a VE strain level of up to 170 $\mu\text{m}/\text{m}$. This proves that the results obtained by the proposed algorithm are consistent with the assumption made and therefore reliable.

On the occasion of this study, the effect of the stress level on the VP strain evolving in an SSCR test was also revealed. As can be seen in Figure 11(a), a higher stress level led to a higher permanent strain in the sample and vice-versa. Moreover, the VP strains were likely to linearly increase with the stress level ranging from 11.90 to 35.71 kPa (see Figure 11(b)). Regarding the proportion of VP strains at 50°C, they accounted for a large part of the total strain, ranging from 30.09% to 41.54%. This implies that a mixture with higher stiffness may have a higher resistance to rutting.

3.2. Multi-stress level creep-recovery test (MSCR)

This subsection is devoted to the examination of the PD behaviour of the mixture subjected to a series of five loading steps with equal durations in ascending or descending order as shown in Figures 12(a,b). These simulated loading histories represent the effects of a series of vehicles in two scenarios: (i) The heaviest passing first, followed by the next heaviest ones, and (ii) the lightest one passing first, followed by the next lightest ones. The result of this study will respond to the question in which scenario

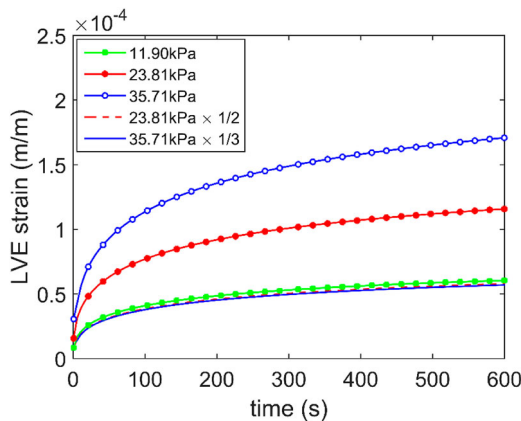


Figure 10. Check of linearity for VE strains at 50°C.

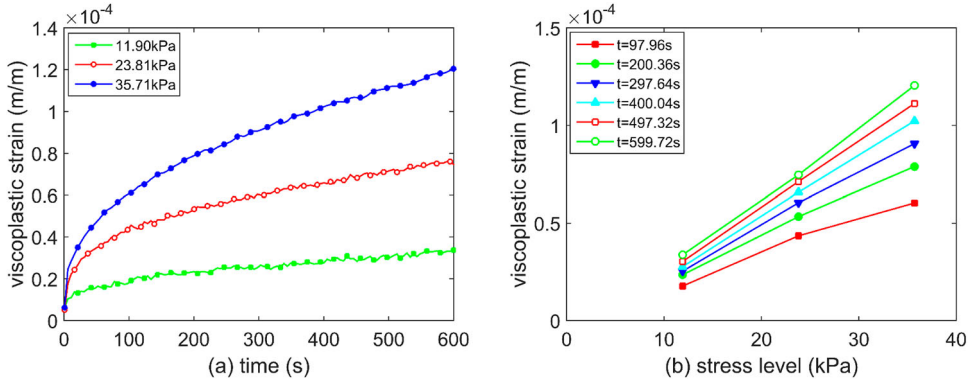


Figure 11. Effect of the stress level on the VP strain determined with the proposed method at 50°C.

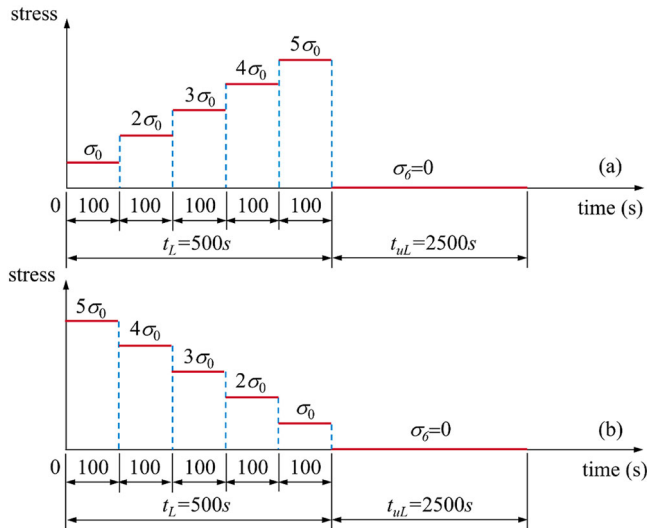


Figure 12. Loading patterns in (a) the increasing MSCR test and (b) decreasing MSCR test (not drawn to scale).

the material will accumulate more PD, which has not been yet adequately studied. In fact, studies on resistance to rutting of mastics and binders using increasing MSCR tests are abundant (e.g. Lagos-Varas et al., 2022; Laukkanen et al., 2015; Zahid et al., 2016), while very little research has been done with asphalt mixtures undergoing increasing MSCR tests (Bai et al., 2014) or decreasing MSCR tests. In addition, the proposed test conditions will allow us to verify the efficiency of the GCR test in dealing with more complicated loading histories.

Since the applied stress was not constant during MSCR tests, the linearity of the stress-VE strain relationship was ensured by choosing the maximal value of the stress such that the maximal value of the VE strain was small enough (lower than $170 \mu m/m$). All test replicates were performed on the same sample (#3) with the maximal stress equal to $5\sigma_0 = 40.82 kPa$. The total number of test replicates was 6, of which three test replicates were performed in the increasing mode (coded test #1 to #3), and three were performed in the decreasing mode (coded test #4 to #6). The two types of tests were alternately performed so as to minimise the effect of the cumulative permanent strain on the behaviour of the sample, i.e. the order of tests is #1, #4, #2, #5, #3, and #6. The loading period was divided into 5 equidistant steps of 100s, and the duration of the recovery period was not predefined a priori. In reality, after the loading period had finished, the sample was left to completely recover the reversible

strain so as to measure the ‘true value’ of the VP strain in the sample. However, only the data of the recovery period in the first 2500s were retained for the back-calculation algorithm as suggested in the previous subsection.

3.2.1. Increasing multi-stress level creep-recovery test

Figure 13(a) presents the measured strain and the predicted VE strain obtained in the increasing MSCR test replicate #1. Both the measured and the VE strain have the same trend of evolution – namely, their values steadily increased during the loading period and then recovered during the resting period. The VP strain was found to increase at a nearly constant rate in the loading period yet almost remain unchanged during the resting period (see Figure 13(b)), as expected. It can be demonstrated that the VP strain in the first increasing MSCR test replicate can be predicted with a high R^2 value by the piece-wise linear function: $\varepsilon_{vp}^1 = 1.81 \times 10^{-7}t + 4.09 \times 10^{-6}$ for $t \leq 500s$, and $\varepsilon_{vp}^1 = 9.31 \times 10^{-5}$ for $t > 500s$.

The back-calculated CCs were compared to the CCs interconverted from DM data so as to evaluate the reliability of the proposed method (Figure 14(a)). All the obtained CC curves are in relatively good accordance – namely, the relative difference between any CC curve and the average curve is less than 7.19%. This certainly leads to good agreement between the predicted VE strains, as can be seen in Figure 14(b). In terms of accuracy, the average curve of the back-calculated CCs differs approximately 17.04% from the interconverted CCs of sample #1. This rather large difference may stem from the heterogeneity of the mixture that induces the inter-sample variations and therefore is not entirely a lack of accuracy. However, an inter-sample variation level of $< 17.04\%$ is still accepted in pavement engineering.

Interestingly, the VE strain in test replicate #1 is the highest, followed by that in test replicate #2, and the VE strain in test replicate #3 is the smallest. Thus, it is obvious that the order of tests has some influence on the VE properties of the mixture to the extent that the mixture may gain extra stiffness. This can be explained by the fact that the aggregate in the mixtures gets closer and closer due to the increase of permanent contraction strain. As a result, the interlock was enhanced and thus the mixture was hardened. In all test replicates, the VE strain did not exceed $170 \mu m/m$, and thus the linearity of the stress-VE strain relationship was ensured.

Figure 15 shows the VP strains determined from the data of all increasing MSCR test replicates. All VP strains demonstrate a nearly linear relationship with the loading time. In fact, the development of VP strains in test replicates #2 and #3 can also be predicted with high R^2 values by the linear functions: $\varepsilon_{vp}^2 = 1.66 \times 10^{-7}t + 6.06 \times 10^{-6}$, $\varepsilon_{vp}^3 = 1.52 \times 10^{-7}t + 2.40 \times 10^{-6}$ respectively.

It should be noted that the PD behaviour of the material is initially not the same in all test replicates; therefore, the obtained VP strains basically cannot be compared with each other. However, if the

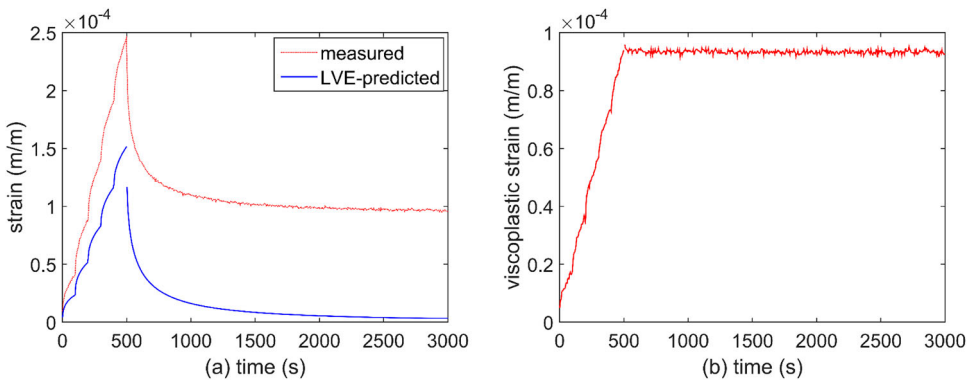


Figure 13. Results of increasing MSCR test replicate #1 at 50°C, (a) Predicted VE and measured strains, (b) Computed VP strain with time.

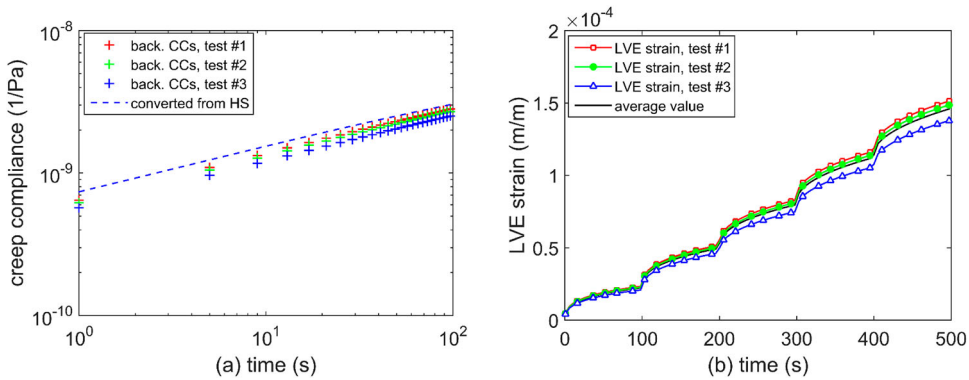


Figure 14. Back-calculation results of all decreasing MSCR test replicates at 50°C, (a) back-calculated CCs, (b) Predicted VE strains.

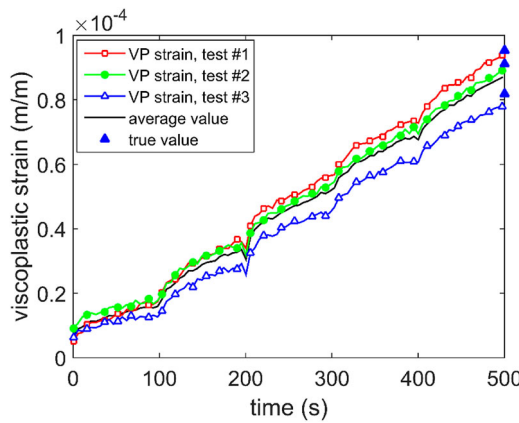


Figure 15. Evolution of VP strains at 50°C in sample #3 during all increasing MSCR test replicates.

change in the resistance to PD of the material from one test replicate to another one is not too large, the comparison can be considered meaningful. In fact, the relative difference between any curve of VP strain and the average curve is lower than 10.91%; thus the obtained VP strains can be considered repeatable. In addition, the computed VP strains differ less than 4.78% from the ‘true values’. Therefore, the proposed method is able to determine reliable VP strain values even with a more complicated loading history.

As mentioned above, the obtained VP strains were affected by the order of testing, or more precisely, by the amount of PD gradually accumulated in the sample. Figure 15 clearly shows a reduction of VP strain with the order of test replicate – namely the VP strain at 500s in test replicate #1 is the highest ($93.82 \mu\text{m/m}$), followed by that in test replicate #2 ($89.15 \mu\text{m/m}$), and that in test replicate #3 is the smallest ($77.94 \mu\text{m/m}$). Thus, the accumulation of the contraction irreversible strain not only increased the stiffness but also reduced the rutting potential of the material.

3.2.2. Decreasing multi-stress level recovery test

Figures 16(a,b) show the measured strain in test replicate #4 caused by the input stress consisting of five levels in descending order: 40.82, 32.66, 24.49, 16.33, and 8.16 kPa . As displayed in the figure, the response strain of the sample was rather complicated and may be summarised as follows.

In the first loading interval, the response strain drastically increased as a result of the stress that was applied at the highest level. After that, the response strain rapidly recovered at the beginning of the

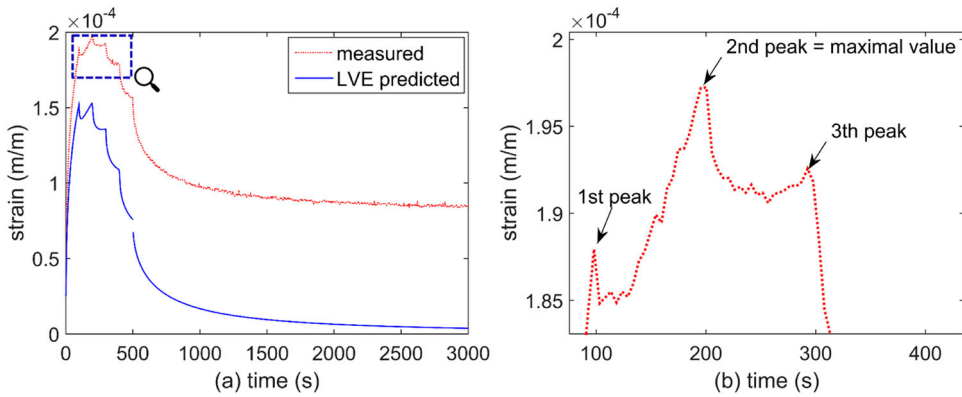


Figure 16. Results of decreasing MSCR test replicate #1 at 50°C, (a) Predicted VE and measured strains, (b) Zoom on the measured strain at the peak values.

second loading interval due to the sudden reduction of 8.16 kPa from the current stress level. However, the recovery of the response strain lasted only in 5–10 s because the increasing rate of VP strain quickly dominated the decreasing rate. Since that moment, the response strain continuously went up until the end of the second loading interval. In the third loading interval, the applied stress level was again reduced by an amount of 8.16 kPa, and therefore the strain once more started to recover for approximately 60s before resuming the increasing phase. Nevertheless, the overall tendency of the strain in this loading interval was to recover the reversible strain. Consequently, the measured strain has three peak values at $t = 100, 200$, and 300 s, among which the one at 200 s is the maximum ($197.24 \mu\text{m/m}$). Since the onset of the fourth loading period, the measured strain has been in a steadily decreasing phase. The predicted VE strain also has the same trend of evolution as the total strain but to a lesser extent (see Figure 16(a)). It also had three peak values at $t = 100, 200$, and 300 s and reached its maximum at the second peak ($153.01 \mu\text{m/m}$).

Interestingly, the VP strain showed a steady increase with increasing loading time and almost remained constant during the resting period even though the measured strain clearly showed a decreasing phase in the last three loading intervals (see Figure 17). This may be explained by the fact that the sample was not free of stress during all loading intervals, causing an irreversible strain rate in the sample and making the VP strain to grow. Additionally, compared to the total strain, the VE strain rose during the first two loading intervals with a lower overall increasing rate, while the VE strain plunged during the last three loading intervals with a higher rate. Hence, the difference between these two strains certainly results in a quantity that steadily rises with an increase in the loading time.

Like the increasing MSCR test, the overall law of development of the VP strain in the decreasing MSCR test is likely to be linear in the loading period and almost constant in the resting period, which can be predicted with a high R^2 value by the piece-wise linear functions: $\varepsilon_{vp}^4 = 1.20 \times 10^{-7}t + 2.41 \times 10^{-5}$ for $t \leq 500$ s and $\varepsilon_{vp}^4 = 8.05 \times 10^{-5}$ for $t > 500$ s. When compared to the increasing MSCR test, the VP strains in the decreasing MSCR witnessed a much higher jump at the beginning of the test (approximately 4–5 times larger) and a lower growth rate (approximately 34% lower). This is due to the fact that the initial stress level was higher, and the value of the stress declined with loading time.

The CCs obtained from all three decreasing MSCR test replicates were back-calculated and are shown in Figure 18(a). All the CC curves agree relatively well with each other – namely, the relative difference between any curve of CC and the average curve is lower than 8.18%. As a result, there was also relatively good agreement between all the predicted VE strains (see Figure 18(b)). In terms of accuracy, the average curve of the back-calculated CCs differs approximately 10.11% from the interconverted CCs of sample #1, which is an acceptable value.

Similar to the increasing MSCR test, the order of testing also influences the VE properties of the mixture. In fact, the VE strain in test replicate #4 is the highest, followed by that in test replicate #5, and

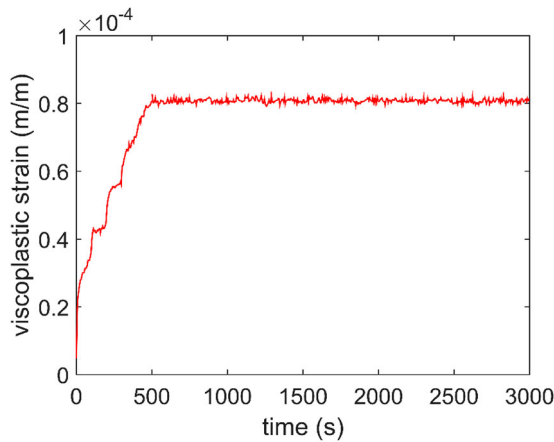


Figure 17. VP strain in decreasing MSCR test replicate #1 at 50°C.

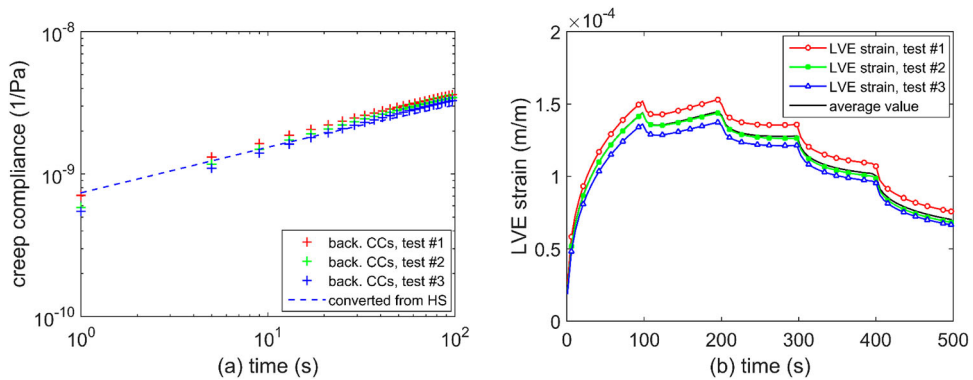


Figure 18. Back-calculation results of all decreasing MSCR test replicates at 50°C, (a) Back-calculated CCs, (b) Predicted VE strains.

the VE strain in test replicate #6 is the smallest (see Figure 18(b)). As discussed above, the aggregate was rearranged under the effects of the cumulative permanent contraction strain, and therefore the mixture hardened. In all decreasing MSCR test replicates, the maximal value of the VE strain did not exceed $170 \mu\text{m/m}$; therefore, a linear stress-VE strain relationship was also ensured.

Figure 19 shows the VP strains determined from the data of all decreasing MSCR test replicates. The relative difference between any curve of VP strain and the average curve is lower than 4.35%. In addition, the computed VP strains differ less than 4.41% from the 'true values'. Therefore, the proposed method is capable of determining reliable VP strains even with a complicated loading condition like that in the decreasing MSCR test.

The VP strain at 500s in test replicate #4 is the highest ($80.53 \mu\text{m/m}$), followed by that in test replicate #5 ($75.62 \mu\text{m/m}$), and that in test replicate #6 is the smallest ($72.38 \mu\text{m/m}$), proving that the cumulative contraction permanent strain had an impact on the resistance to rutting as already observed in the increasing MSCR tests. However, the overall variations of VP strains obtained from all test replicates are still small ($< 4.35\%$); therefore, the bias could be considered small and did not affect much the agreement between the obtained results.

It remains to us to respond to the question raised in the introduction of the MSCR test. Based on the results obtained in the increasing and decreasing MSCR test replicates, one can conclude that the loading pattern does affect the amount of VP strain accumulated in the sample. In fact, the average

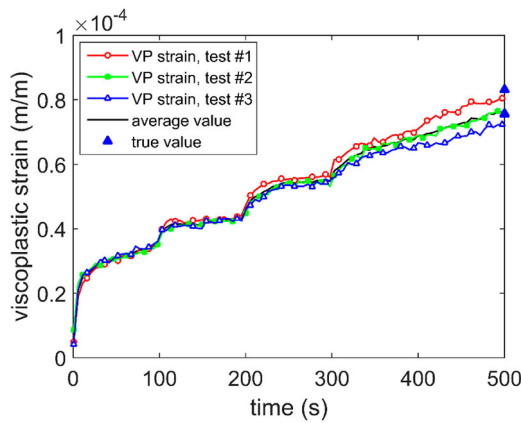


Figure 19. Evolution of VP strains at 50°C in sample #3 during all decreasing MSCR test replicates.

value of VP strains at the end of the loading period obtained in the decreasing MSCR test is 76.18 $\mu\text{m/m}$ while that obtained in the increasing MSCR test is 86.97 $\mu\text{m/m}$, which is 14.16% higher.

4. Conclusions

In this paper, a generalised version of the creep-recovery test procedure has been proposed to examine the permanent deformation of asphalt mixtures exerted by any complex loading history in the time domain. The results of the generalised creep-recovery test were successfully used to characterise both the viscoelastic and permanent deformation behaviour of the material at 50°C without conducting dynamic modulus tests. Owing to the present back-calculation algorithm, the creep compliances were determined with a good level of accuracy and repeatability, based on which the viscoplastic strains could be accurately determined even with short recovery periods. The main findings of this work can be drawn as follows:

- The AACC function derived from the Huet-Sayegh model outperforms the Prony series CC function when used to fit the recovery compliance data. The CCs obtained with the latter function deviated far from the exact CCs, especially at low loading times, while those obtained with the creep function derived from the Huet-Sayegh model showed a good agreement with the exact CCs.
- The duration of the recovery period has a significant impact on the directly measured VP strains at 50°C; however, it does not affect much the back-calculated VP. In fact, a resting time equal to 2–3 times the loading time could lead to an accuracy level of roughly 3% for the back-calculated VP strains. Therefore, a ratio of resting time to loading time ranging from 2 to 5 was sufficient for the back-calculation algorithm to give reliable results for tests at 50°C. Conversely, a much longer resting period was needed for the direct measurement to achieve good results at the same testing temperature, at least 7 times the loading time.
- At a high temperature (50°C), permanent strains account for an important part of the total strains, which suggests that a mixture with higher stiffness may have a higher resistance to rutting.
- When the resting time equals 5 times the loading time, the relative errors of the VP strain are lower than 4.78% while any curve of VP strain differs < 10.91% from the average curve, which proves that the back-calculation algorithm is reliable at the high testing temperature (50°C).
- The loading history does affect the development of VP strain. With the same number of loading intervals, duration of loading period, and maximal level of the applied stress, the VP strain obtained in the increasing MSCR test was larger than that obtained in the decreasing MSCR test, approximately 14.16% higher in the conditions studied.

According to the obtained results, the proposed test procedure allows determining with good accuracy and repeatability the pure permanent deformation of asphalt mixtures. Therefore, the results can be used to calibrate predictive models as well as to evaluate the rutting potential of asphalt mixtures. For instance, the present increasing MSCR test may be extended to be used as an accelerated test for evaluating the resistance to rutting of asphalt mixtures like the standardised MSCR test already developed for asphalt binders.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

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Verification of time-temperature superposition principle for viscoplastic strains of asphalt mixtures obtained from creep-recovery tests in the low-strain regime

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Verification of time-temperature superposition principle for viscoplastic strains of asphalt mixtures obtained from creep-recovery tests in the low-strain regime

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ABSTRACT

The creep compliances, total, and viscoplastic (VP) strains of various asphalt mixtures were determined from creep-recovery tests at different temperatures to construct their master curves. In order to ensure the reliability of the obtained results, the agreement between quantities in the time and frequency domain as well as sources of bias in testing were investigated. A viscoelastic and a VP model were used to respectively characterise the reversible and irreversible deformation of the materials. The behaviour of the specimens exerted by a constant stress at various temperatures were then predicted using independent values of the viscoelastic and VP reduced times. Owing to the good agreement between prediction and measurement, the time-temperature superposition principle in the VP domain was verified. In general, the shift factors were uncorrelated with each other. However, at elevated temperatures, the gaps between them became unimportant, enabling the extension of viscoelastic shift factors into the VP domain.

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1. Introduction

Rutting, or permanent deformation (PD), is among the most common distresses of flexible pavement, resulting in a severe decrease in traffic safety. According to many studies (Garba, 2002; Khan et al., 2013; Xu & Huang, 2012), the main mechanism causing PD is the accumulation of viscoplastic (VP) strain of asphalt mixture layers under the action of loading and climatic conditions. Therefore, understanding well the PD behaviour of asphalt mixtures is crucial for properly predicting the evolution of the rut depth, based on which appropriate materials could be chosen or designed.

Hamburg Wheel Tracking test (HWT) has been extensively used to characterise the resistance to PD of asphalt mixtures (Hua, 2000; Nahi et al., 2011; Nguyen, 2017). In such a test, the deflection at the end of each load cycle is considered as the rut depth regardless of the contribution of viscoelastic (VE) deformation. Therefore, the measured rut depth in an HWT test is not the one caused by the pure PD of the material (Tran et al., 2023). In addition, the stress and strain fields in the sample during an HWT test are not homogeneous, causing difficulties in the investigation of VP strain evolution.

Different from HWT test, repeated load permanent deformation (RLPD) tests conducted in uniaxial or triaxial compression of prismatic samples ensure uniformly distributed stress and strain fields

inside the sample. Consequently, plenty of studies have used RLPD tests to investigate the development of VP strain in asphalt samples (e.g. Dong et al., 2018; Fang et al., 2017; Isailovic et al., 2016; Shume, 2002). The common loading shapes used in RLPD tests are haversine or multistep functions with or without resting periods. However, the VE strain needs an extremely long time to entirely recover; thus, the VP strain measured in RLPD tests is not the pure VP strain yet includes a part of the VE strain that has not yet recovered despite a rather long resting period has been applied (Tran et al., 2023).

Another drawback of RLPD tests is that VE characteristics cannot be determined from the results of the test due to the short loading time applied and is thus commonly neglected. In some studies (e.g. Darabi et al., 2012; Saadeh et al., 2007), extra dynamic modulus (DM) tests were performed to characterise the VE properties of the material, from which the pure permanent deformation can be extracted. Such a two-stage characterisation certainly requires much more testing time, and more importantly, accumulated VP strain during the preceding test will cause biases in the results obtained in the subsequent one. In the case two different samples are used (one for RLPD and one for dynamic modulus test), inconsistency between the VE properties of the two samples may give rise to negative values of the VP strains obtained at a low number of loading cycles.

In contrast, the creep-recovery (CR) test is able to characterise simultaneously the VE and VP behaviour in just one single run (Drescher et al., 1993). In fact, the VE creep compliance (CC) can be back-calculated based on the data of the resting period while the VP strain can be computed from the measured strain in the loading period (Bai et al., 2014; Chen et al., 2017; Tran et al., 2023). The CCs determined from CR tests were demonstrated to be in full accord with those interconverted from dynamic modulus data (Nguyen et al., 2021b; Tran et al., 2023). Thus, the VP strains obtained from CR tests are indeed the pure irrecoverable strains.

In the linear viscoelastic (LVE) domain, the response of asphalt mixtures caused by a longer duration of the load and at a lower temperature is equivalent to that caused by a shorter loading time and at a higher temperature, called time-temperature superposition principle (TTSP) (Ferry, 1980; Tschoegl, 1989). It is well-known that asphalt mixtures are thermorheologically simple materials in the LVE domain. Consequently, the master curve of LVE properties can be constructed by using horizontal shifting techniques (Forough et al., 2014; Qian et al., 2008). Similarly, asphalt mixtures were demonstrated to remain thermorheologically simple in the nonlinear viscoelastic region with or without growing damage, and fatigue behaviour (Boussabnia et al., 2023; Bradai et al., 2022; Chehab et al., 2002; Nguyen et al., 2009; Nguyen et al., 2013b; Nguyen et al., 2013a; Schapery, 1997; Schapery, 1999; Schwartz et al., 2002).

Zhao and Kim (2003) conducted constant strain rate tests at various temperatures to show that TTSP still holds in the VP domain with growing damage by constructing stress-log reduced-time master curves. With this approach, the master curves of VP strain, or the curve of VP strain caused by the same stress level in a wide range of time of interest, were not able to be constructed. Moreover, RLPD tests with the same loading conditions at 25, 40, and 55°C were additionally performed by Zhao and Kim (2003). The testing times were chosen such that the reduced times calculated with VE shift factors were the same. The strains obtained at different temperatures were then all plotted versus the reduced time to demonstrate the agreement, hence the validity of TTSP in the VP domain. The approach was then adopted by Yun et al. (2010) to validate TTSP in the VP domain in considering the effect of the confining stress. Later, in an attempt to build the master curve of PD over a wide range of the reduced time, Khosravifar et al. (2014) performed RLPD tests at three temperatures per mixture with the same testing time and the obtained PD at each temperature was shifted using the shift factors determined from dynamic modulus tests. Since the steps between the investigating temperatures were rather large, there were no overlap portions in the master curves of the PD obtained, and thus the characteristics of the master curve were not assessed. Recently, Cao and Kim (2022) proposed a method to determine VP shift factors by determining the value of a temperature-related parameter in the prediction of PD obtained from triaxial RLPD tests. However, the construction of the VP strain master curve was not considered by the authors. In the above-mentioned works, the application of TTSP in the VP domain

was partially demonstrated. Either the appropriate VP shift factors or the master curves of VP strains have not been yet investigated.

Therefore, the objective of this work is to adequately verify the applicability of TTSP in the VP domain within the low-strain regime. The reason for limiting the strain amplitude is to ensure that the LVE theory is valid and the LVE properties in both time and frequency domains can be accurately determined using fundamental procedures, based on which the reliability of the obtained results in the VP domain can be assessed. The specific objectives of this work are defined as follows:

- investigating the pure VP strains in the low-strain regime of a variety of asphalt mixtures caused by several stress levels and temperatures using CR tests;
- developing new theoretical formulations to quantitatively estimate the influences of various factors causing biases in the obtained results;
- examining the extension of existing LVE shifting techniques into the total and VP strains and developing a new shifting method to reliably determine the shift factors;
- analysing the properties of the obtained master curves and their corresponding shift factors to theoretically demonstrate the applicability of TTSP in the VP domain;
- applying a simple VP model for predicting the irreversible strains of all studied mixtures to demonstrate the efficiency of the master curve of VP strains and the companion VP shift factors in characterising and predicting the PD behaviour of asphalt mixtures, based on which the TTSP in the VP domain can be experimentally validated.

2. Materials and methods

2.1. Materials and samples preparation

In this study, the application of TTSP in the LVE and VP domain of three asphalt mixtures including a conventional mixture (AC), a polymer-modified bitumen (PMB) asphalt mixture, and a highly modified asphalt mixture (HiMA) was examined. The AC mixture was prepared with a neat bitumen of grade PG 64 and the PMB mixture was produced with a polymer-modified bitumen. The PMB was a blend of the neat bitumen and 4% ÷ 5% (in total weight) of styrene–butadiene–styrene (SBS) such that its performance grade attained PG 76. In order to prepare the HiMA mixture, a highly modified bitumen of PG 94 was used. It was obtained by modifying the PG 64 bitumen with approximately 2% (in total weight) of SBS and a suitable amount of high modulus additive. All mixtures were made of andesite aggregates, which were collected in the south of Vietnam. The nominal maximum aggregate size of all mixtures is 12.5 mm and the proportions of all aggregates were chosen similar to those in (Nguyen et al., 2021a). The fundamental properties of all mixtures were determined from three samples and their mean values are reported in Table 1.

In order to prepare specimens for tests, the mixtures were compacted by a roller sector compactor at 145, 160, and 175°C to obtain three plates of AC, PMB, and HiMA mixtures, respectively. The dimensions of the plates were $0.32 \times 0.26 \times 0.08 \text{ m}^3$. After cooling down to room temperature, the plates were

Table 1. Engineering properties of the studied asphalt mixtures.

Engineering properties	Standards	Results		
		AC	PMB	HiMA
Optimum asphalt content (%)		4.8	5.0	5.0
Maximum specific gravity	ASTM D2041	2.52	2.51	2.51
Bulk specific gravity	ASTM D2726	2.39	2.38	2.36
Air void (%)	ASTM D3203	5.16	5.18	5.98
Void in aggregate (%)		14.02	15.06	15.77
Marshall stability (kN)	ASTM D6927	10.60	15.50	19.00
Marshall flow (mm)	ASTM D6927	3.50	3.40	3.10
Indirect tensile strength at 25 °C (IDTS) (MPa)	ASTM D6931	0.98	1.24	1.97



Figure 1. Preparation of samples for AC, PMB, and HiMA mixtures.

sawed into three prismatic samples per mixture with dimensions: $0.07 \times 0.07 \times 0.15 \text{ m}^3$ as shown in Figure 1. It is worth noting that the discrepancy of the air void content between the prismatic specimens was little thanks to the strict control of the mixing and compaction temperature. The standard deviations (SD) of the air void contents were 0.34%, 0.22%, and 0.10% for AC, PMB, and HiMA mixtures, respectively. The corresponding relative standard deviations (RSD) were 6.56%, 4.24%, and 1.69% of the mean values.

2.2. Test set-up

The set-up of the CR test is described in Figure 2, in which the thermal chamber was used to maintain a constant temperature, two LVDTs (linear variable displacement transducers) were used to measure the strain, and the control and data acquisition system (CDAS) was responsible for controlling the applied

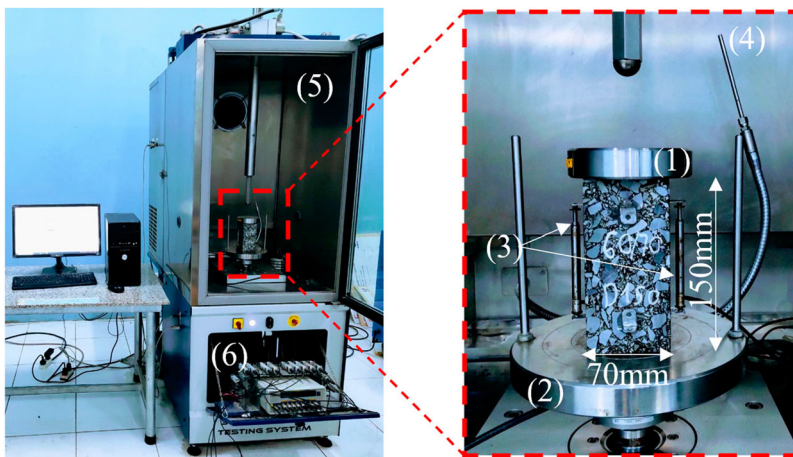


Figure 2. Test set-up for the CR test: (1) Upper load platen, (2) Lower load platen, (3) Displacement transducer, (4) Control temperature sensor, (5) Thermal chamber, (6) CDAS.

force and acquiring data during testing. It is worth mentioning that the actuator orientation was bottom loading. In order to ensure a homogenous temperature field, the specimen was left in the thermal chamber for at least four hours before starting a test.

2.3. Determination of VP strain using single stress level creep-recovery (SSCR) test

In this study, the VP strain was calculated using the algorithm proposed by Tran et al. (2023), which is comprised of three steps: (i) determination of creep compliance function, (ii) prediction of viscoelastic (VE) strain, and (iii) calculation of VP strain, summarised as follows. Let the specimen be at zero-strain and zero-stress conditions prior to the application of a single-step function as illustrated in Figure 3a. With such a stress history $\sigma(t)$, the typical response strain $\varepsilon(t)$ in the specimen can be classified into two phases: a developing one and a recovery one where a part of the strain is recovered once the stress is removed as illustrated in Figure 3b. Therefore, it has been assumed in many studies that the response strain, or the total strain, in an SSCR test is the summation of VE and VP strains (e.g. Alammie et al., 2022; Bai et al., 2014; Lai & Anderson, 1973; Levenberg, 2006; Perl et al., 1983; Sun & Zhu, 2013):

$$\varepsilon_T(t) = \varepsilon_{VE}(t) + \varepsilon_{VP}(t) \quad (1)$$

It should be noted that damage strain may occur under tensile stress or very high compressive stress conditions. Thus, the applied compressive stress in this work was limited to being small enough so that damage could be neglected. Within the small VE strain assumption, the Boltzmann superposition principle is applied to compute the VE strain:

$$\varepsilon_{VE}(t) = D(t - t_0)\sigma_1, \text{ for } t_0 < t \leq t_1 \quad (2)$$

$$\varepsilon_{VE}(t) = D(t - t_0)\sigma_1 - D(t - t_1)\sigma_1, \text{ for } t > t_1 \quad (3)$$

where $D(t)$ is the CC function being served as the unknown LVE properties.

Since the stress during the resting period is equal to zero, one can assume that VP strain does not evolve during this interval of time:

$$\varepsilon_{VP}(t) = \varepsilon_{VP}(t_1), \text{ for } t > t_1. \quad (4)$$

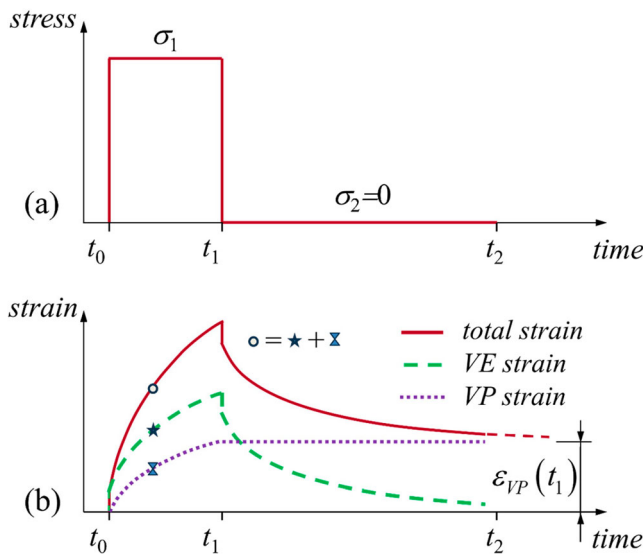


Figure 3. Illustration of the stress history (a) and response strain (b) in an SSCR test.

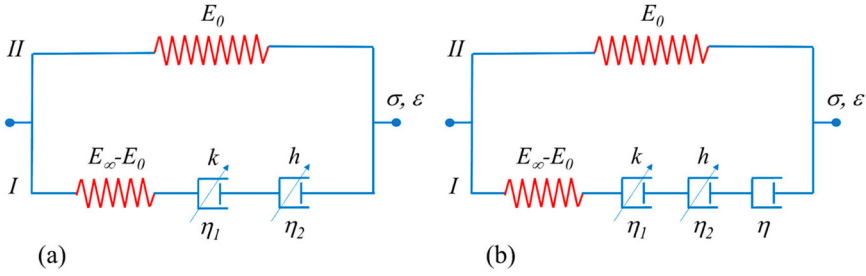


Figure 4. Illustration of Huet-Sayegh (a) and 2S2P1D (b) models.

Consequently, one can easily demonstrate that (Tran et al., 2023):

$$\frac{\varepsilon_T(t_1) - \varepsilon_T(t)}{\sigma_1} = D(t_1 - t_0) - D(t - t_0) + D(t - t_1), \text{ for } t > t_1, \quad (5)$$

wherein the VP strain is eliminated from the equation. With a chosen form function of $D(t)$, the functional Equation (5) can be solved in the sense of least squares. Owing to the excellence in fitting LVE behaviour of asphalt mixtures (Cardona et al., 2015; Hornych et al., 2002; Nguyen et al., 2021a; Olard & Di Benedetto, 2003; Pronk, 2005), the analytical approximate creep compliance (AACC) functions derived from Huet-Sayegh (HS) and 2S2P1D models were suggested to be used as a solution to Equation (5) (Nguyen et al., 2021b; Tran et al., 2023).

The HS and 2S2P1D models include one spring related in parallel with a series of one spring and two fractional dashpots (and a linear dashpot) as illustrated in Figure 4. Their complex moduli were given in (Olard & Di Benedetto, 2003; Pronk, 2005; Sayegh, 1965) and are presented in Equation (6):

$$E_{HS}^*(i\omega) = E_0 + \frac{E_\infty - E_0}{1 + \delta(i\omega\tau(T))^{-k} + (i\omega\tau(T))^{-h}}$$

$$E_{2S2P1D}^*(i\omega) = E_0 + \frac{E_\infty - E_0}{1 + \delta(i\omega\tau(T))^{-k} + (i\omega\tau(T))^{-h} + (i\omega\beta\tau(T))^{-1}}, \quad (6)$$

where ω (rad/s) is angular frequency; δ , $0 < k < h < 1$ are model parameters; E_0, E_∞ are the value of the complex modulus when $\omega\tau \rightarrow 0$ and $\omega\tau \rightarrow \infty$, respectively. The AACC function was derived from the complex modulus by using the analytical approximate method of Schapery and Park (1999):

$$D\left(t = \frac{1}{\omega}\right) = \frac{\sin(n_s\pi)}{n_s\pi} \cdot \frac{\lambda'(\omega)}{E'(\omega)}; \lambda'(\omega) = \Gamma(1 - n_s) \cos\left(n_s \frac{\pi}{2}\right), \quad (7)$$

where $\Gamma(\cdot)$ is the Gamma function, $E'(\omega)$ is the storage modulus, and n_s is the local slope coefficient of the curve of $\log(E'(\omega))$ versus $\log(\omega)$, which can be explicitly calculated by a closed-form formula as given in (Nguyen et al., 2022).

Once the CC function was determined, the pure VE strain can be predicted using Equations (2)-(3) and the pure VP strain ($\varepsilon_{VP}(t)$) is computed by eliminating the VE strain from the total strain:

$$\varepsilon_{VP}(t) = \varepsilon_T(t) - \varepsilon_{VE}(t). \quad (8)$$

3. Experimental results and analysis

This section presents the results obtained from a variety of SSCR tests performed at different stress levels and temperatures. The stress levels and the temperatures were carefully chosen such that the

Table 2. Summary of the experimental testing programme.

Test No.	Sample	Type of test	Temperature (°C)	Stress σ_1 (kPa)	Purpose
AC-1,2,3,4	1.1	SSCR	17.5; 25; 32.5; 40	23.81	check TTSP
AC-5,6,7,8	1.2	SSCR	17.5; 25; 32.5; 40	35.71	check TTSP
AC-9	1.2	DM	—	—	check time-frequency agreement
AC-10,11,12,13	1.3	SSCR	17.5; 25; 32.5; 40	53.57	check TTSP
AC-14,15,16	1.3	SSCR	25	53.57	check repeatability
AC-17,18,19	1.3	SSCR	32.5		
AC-20,21,22	1.3	SSCR	40		
PMB-1,2,3,4	2.1	SSCR	27.5; 35; 42.5; 50	23.81	check TTSP
PMB-5,6,7,8	2.2	SSCR	27.5; 35; 42.5; 50	35.71	check TTSP
PMB-9,10,11,12	2.3	SSCR	27.5; 35; 42.5; 50	53.57	check TTSP
PMB-13	2.3	DM	—	—	check time-frequency agreement
PMB-14,15,16	2.1	SSCR	35	23.81	check repeatability
PMB-17,18,19	2.1	SSCR	42.5		
PMB-20,21,22	2.1	SSCR	50		
HiMA-1,2,3,4	3.1	SSCR	40; 47.5; 55; 62.5	23.81	check TTSP
HiMA-5	3.1	DM	—	—	check time-frequency agreement
HiMA-6,7,8,9	3.2	SSCR	40; 47.5; 55; 62.5	35.71	check TTSP
HiMA-10,11,12,13	3.3	SSCR	40; 47.5; 55; 62.5	53.57	check TTSP
HiMA-14,15,16	3.3	SSCR	47.5	53.57	check repeatability
HiMA-17,18,19	3.3	SSCR	55		
HiMA-20,21,22	3.3	SSCR	62.5		

reversible strain was small enough for the Boltzmann superposition principle to be applied. In this study, three stress levels of 23.81, 35.71, and 53.57 MPa were considered for all mixtures (see Table 2) and each stress level was applied for one single sample per mixture. Since the LVE behaviour of AC, PMB, and HiMA mixture are different, the testing temperatures for each mixture were not the same—namely, 17.5, 25, 32.5, and 40°C for AC mixture; 27.5, 35, 42.5, and 50°C for PMB mixture; and 40, 47.5, 55, and 62.5°C for HiMA mixture. It should be noted that each sample of one mixture was investigated at all testing temperatures as noted above to prevent the effects of the heterogeneity of asphalt mixture on the construction of the master curves of total strain, CC, and VP strain. However, it is obvious that the VP strains accumulated during preceding tests certainly affect the evolution of VP strains of subsequent tests; thus, all tests were conducted from the lowest temperature to the highest one. In addition, the LVDTs were reset to zero before running an SSCR test so that the measured response strain of the specimen in that SSCR test can be considered independent on those obtained in other ones. In terms of testing time, the duration of loading and unloading periods are respectively 150s and 3000s.

The total number of samples was nine, labelled as 1.1, 1.2, and 1.3 for AC mixture; 2.1, 2.2, and 2.3 for PMB mixture and 3.1, 3.2, and 3.3 for HiMA mixture. In subsequent paragraphs, all results including total strain, CCs, and VP strains at single temperatures as well as their master curves will be presented. Furthermore, the results of dynamic modulus tests will also be shown to verify the agreement of LVE properties in the time and frequency domain. The testing programme of this work is summarised in Table 2 to facilitate the reading of this paper.

In this study, the following formula was thoroughly used to compute the relative difference (RD) or relative error (RE) between two data sets:

$$RD, RE = \left[\frac{\sum_j (X_j - Y_j)^2}{\sum_j (Y_j)^2} \right]^{1/2}, \quad (9)$$

where X_j , Y_j are the value of the data set X and Y at discrete point j . In the case of evaluating CCs, the inverted values of the CCs were used instead to avoid possible errors caused by too small values of the CCs.

3.1. Total strains, CCs, and VP strains obtained from SSCR

Figure 5 presents the results of the SSCR test conducted with sample 1.2 at 32.5°C. It can be observed in Figure 5(a) that the AACC function derived from HS model gives rise to a good fitting of the experimental recovery compliance in the unloading period. With the obtained best-fit parameters, the CC was computed and is shown in Figure 5(b). The pure VE strain was also calculated and is presented in Figure 5(c).

It is clear that the VE strain is smaller than the measured strain, which verifies the existence of irrecoverable strain inside the sample even at an intermediate temperature. This proves that using only creep test data (i.e. creep test without a resting period) certainly results in large errors of the VE creep compliance as already mentioned in (Nguyen et al., 2021b). By eliminating the VE strain from the total measured strain, one obtains the VP strains as displayed in Figure 5(d). It can be seen that VP strain continuously increased during the loading period and remained almost constant during the resting period as assumed. Thus, in upcoming paragraphs, just the total and VP strains in the loading periods will be presented.

Figures 6–8 respectively depict the measured strain (total strain) of AC, PMB, and HiMA specimens at different stress levels and temperatures. It is clear that the total strains rose with an increase in stress level or temperature. In addition, AC mixture is the most vulnerable to deformation, followed by PMB mixture, and HiMA mixture has the largest modulus of deformation.

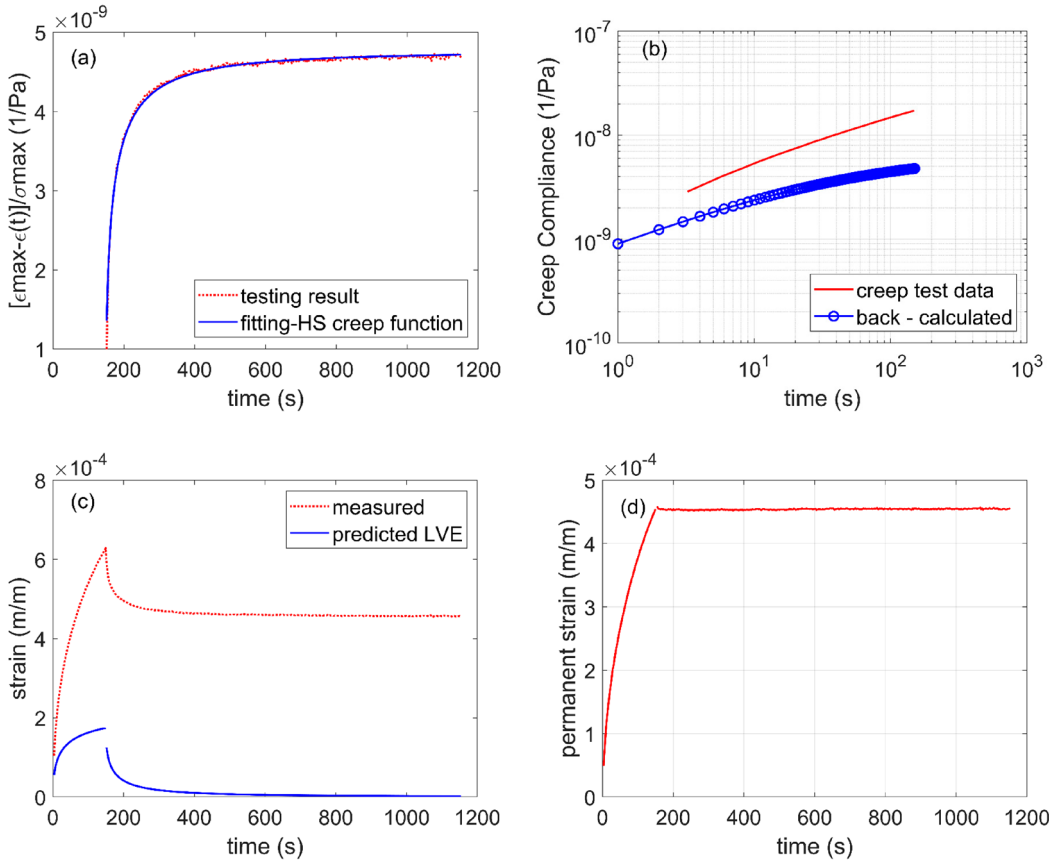


Figure 5. Results of the SSCR test at 32.5°C of AC: (a) Fitting results of the recovery compliance, (b) Back calculated CC, (c) Measured and predicted VE strains, and (d) computed VP strain.

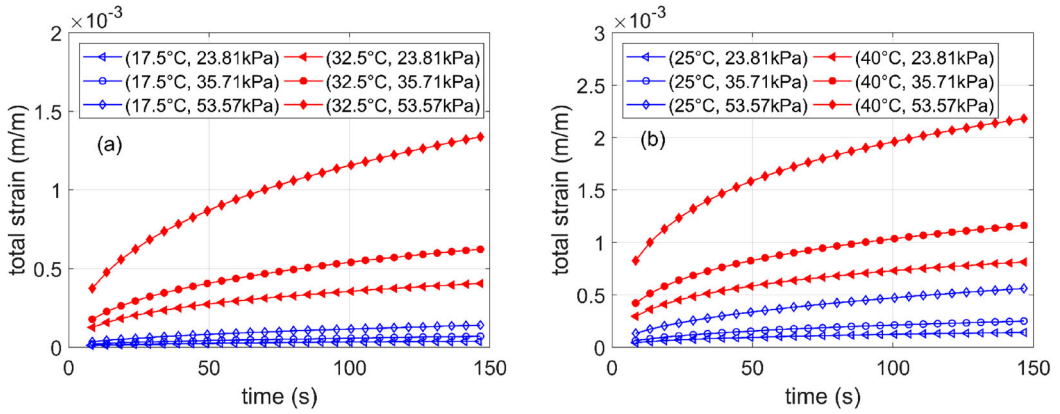


Figure 6. Measured strains (total strains) of AC samples at different temperatures and stress levels (23.81, 35.71, and 53.57kPa): (a) 17.5 and 32.5°C, (b) 25 and 40°C.

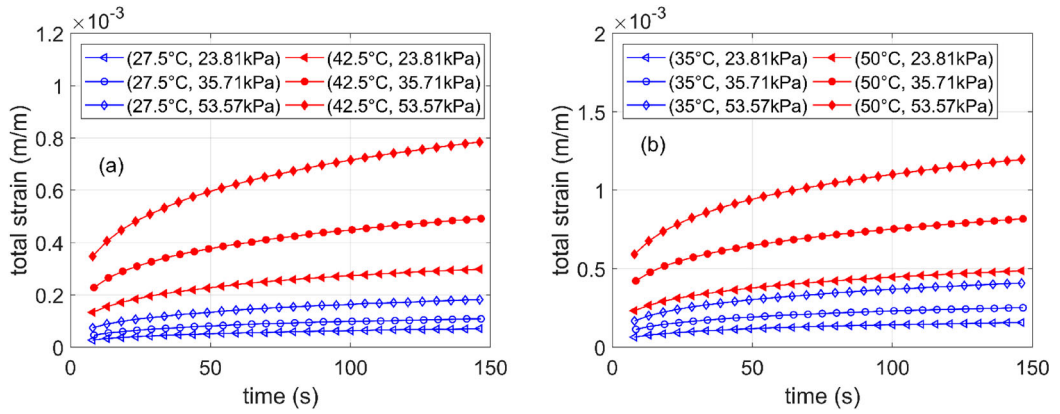


Figure 7. Measured strains (total strains) of PMB samples at different temperatures and stress levels (23.81, 35.71, and 53.57kPa): (a) 27.5 and 42.5°C, (b) 35 and 50°C.

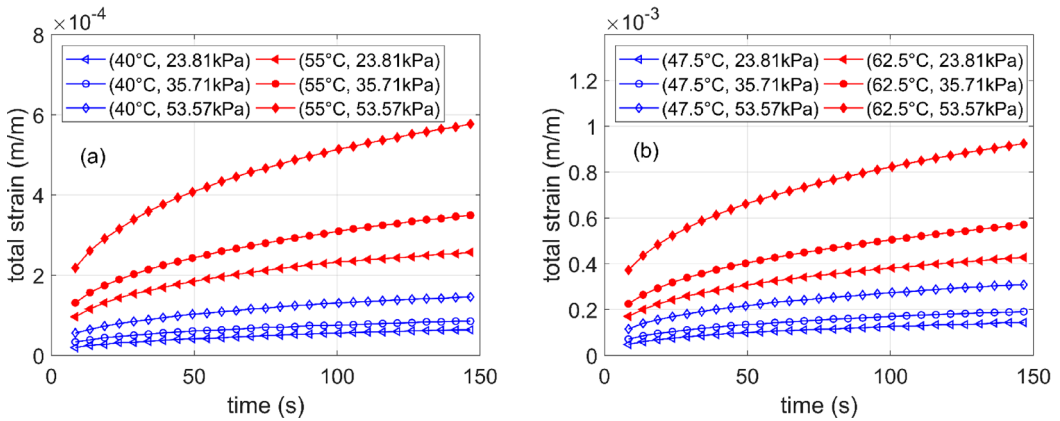


Figure 8. Measured strains (total strains) of HiMA samples at different temperatures and stress levels (23.81, 35.71, and 53.57kPa): (a) 40 and 55°C, (b) 47.5 and 62.5°C.

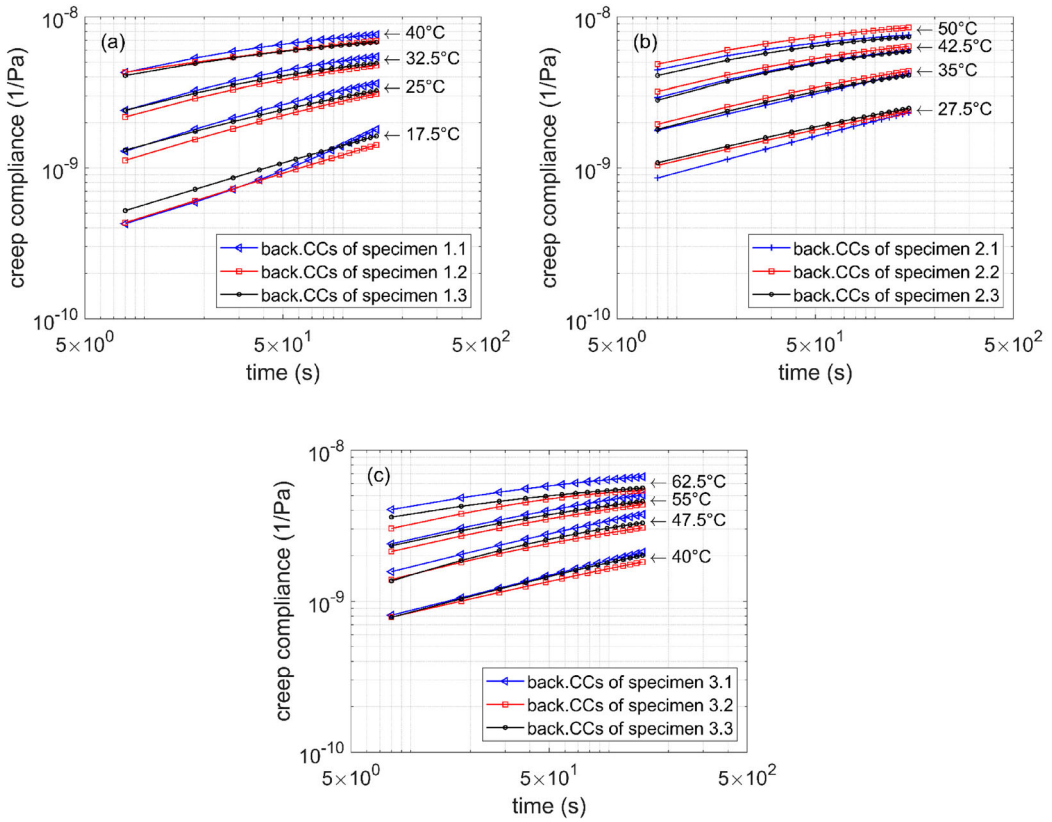


Figure 9. Back calculated creep compliance of asphalt mixtures at different temperatures: (a) AC mixtures, (b) PMB mixtures, and (c) HiMA mixtures.

Figure 9(a), (b), and (c) respectively display the back-calculated CCs of AC, PMB, and HiMA mixtures at different stress levels and temperatures. Once the CCs of all samples were determined, the VP strains of all mixtures were computed and are all presented in Figures 10–12. It should be mentioned that the VP strains of AC mixture at 17.5°C and at 23.81 kPa were not shown. At these testing conditions, the deformation was too small to be accurately captured by the displacement transducers, resulting in unreasonable negative values of the VP strain. It can be observed that the permanent deformation grows with an increase in loading time despite that the applied stress is hold constant. In addition, stress level and temperature also affect the development of VP strain. While stress level has a steady impact on VP strain, temperature dramatically promotes the evolution of VP strain as can be observed in the figures.

3.2. Constructing the master curves of total strain, CC, and VP strain

TTSP says that the LVE property of a thermorheologically simple material at a temperature T_1 and loading time t_1 is equivalent to that at temperature $T_2 > T_1$ and at loading time $t_2 < t_1$, and vice-versa. In other words, the loading time at a lower temperature (T_1) is ‘reduced’ by a factor equal to $a_T = t_2/t_1$ when converted to that at a higher temperature (T_2). Consequently, by choosing an arbitrary temperature as the reference temperature (T_{ref}) and shifting the LVE properties at all temperatures to those at T_{ref} , the master curve of the LVE characteristics can be obtained (Chailleux et al., 2006; Cho, 2016; Tschoegl, 1989). By following the same process, the master curves of the total and pure VP strains were also constructed.

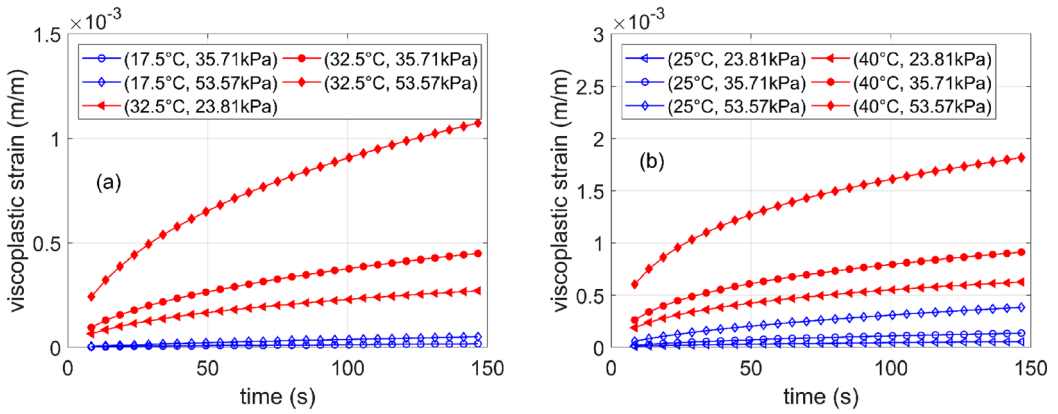


Figure 10. VP strains of AC samples at different temperatures and stress levels (23.81, 35.71, and 53.57 kPa). (a) 17.5 and 32.5°C. (b) 25 and 40°C.

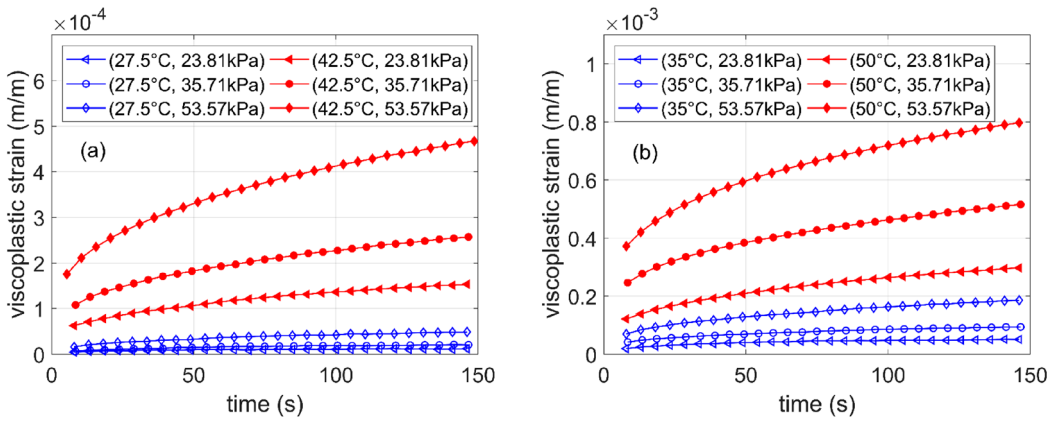


Figure 11. VP strains of PMB samples at different temperatures and stress levels (23.81, 35.71, and 53.57 kPa). (a) 27.5 and 42.5°C. (b) 35 and 50°C.

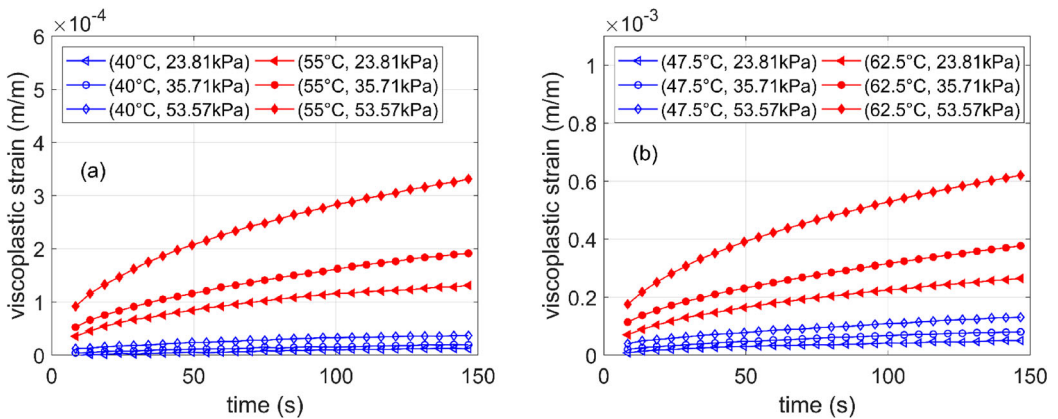


Figure 12. VP strains of HiMA samples at different temperatures and stress levels (23.81, 35.71, and 53.57 kPa). (a) 40 and 55°C. (b) 47.5 and 62.5°C.

3.2.1. Horizontal shifting methods

The shift factors can be determined by simply adjusting adjacent isotherms (Ferry, 1980; Goodrich, 1988); however, the obtained results depend on the experience of the person processing the data (Cho, 2016). In order to ensure the objectivity of the shift factors computed, Honerkamp and Weese (1993) used a nonlinear regression method to calculate the shift factors from the storage modulus data. First, the authors approximated the storage modulus $E'_{T_{ref}}(\omega)$ at the reference temperature by an N -order polynomial of $\log(\omega)$:

$$\log(E'_{T_{ref}}(\omega)) \approx \sum_{k=0}^N a_k \log(\omega)^k \quad (10)$$

Then, the shift factors were determined by minimising the value of:

$$\chi^2 = \sum_{j=1}^{m_T} \sum_{i=1}^{n_j} \left[\log(E'_{T_j}(\omega_i)) - \sum_{k=0}^N a_k (\log(\omega_i) + \log a_T(T_j))^k \right]^2, \quad (11)$$

where $E'_{T_j}(\omega_i)$ is the storage modulus measured at temperature T_j and frequency ω_i ; $a_T(T_j)$ is the shift factor used to convert $E'_{T_j}(\omega_i)$ to that at the reference temperature, m_T is the number of testing temperatures, and n_j is the number of frequencies at temperature j . As suggested by the authors, the order of the polynomial should be chosen in the range of $\sum_{j=1}^{m_T} n_j/20 \leq N \leq \sum_{j=1}^{m_T} n_j/4$. However, according to the work of Cho (2016), the order of the polynomial should be around 10 for the computed shift factors to be independent of the polynomial order. This value is quite high, requiring more data points at one testing temperature so that the approximation by the high-order polynomial could be properly performed. Moreover, in the case where temperature T_j and the reference temperature have a large difference, the determination of shift factors may encounter large errors. This is because the two sets of data do not have an overlap; thus, the calculation of shift factors must be based on the extrapolated data of the storage modulus at the reference temperature. By using a different approach, Chailleux et al. (2006) proposed a formula to calculate the shift factor based on the dynamic modulus and phase angle data at two close temperatures:

$$\log(a_{(T_1, T_2)}) = \frac{\pi}{2} \frac{\log(|E^*(T_1, \omega)|) - \log(|E^*(T_2, \omega)|)}{\bar{\varphi}^{(T_1, T_2)}(\omega)}, \quad (12)$$

where $a_{(T_1, T_2)}$ is the shift factor applied for isotherm T_1 according to temperature T_2 ; $|E^*(T_1, \omega)|$, $|E^*(T_2, \omega)|$ are respectively the dynamic modulus at temperatures T_1, T_2 at a chosen angular frequency ω ; and $\bar{\varphi}^{(T_1, T_2)}(\omega)$ is the average of the two phase angles measured at frequency ω and temperatures T_1, T_2 . The accuracy of the shift factor computed from Equation (12) depends strongly on that of the phase angle which is a quantity more difficult to measure (Phan et al., 2022; Rowe et al., 2009; Zhang et al., 2020). Based only on the dynamic modulus data, Cho (2009) proposed to minimise the sum of distances called 'arc length' formed by the shifted data points obtained at two close temperatures. Based on our observations, the method only works well when the two isotherms have large enough overlaps, or the slope of the curve is not too flat.

Algorithm 1. Algorithm for determining the shift factor

All the above-mentioned methods were originally oriented to be used in the frequency domain. Some of them can be extended into the time domain such as those of Honerkamp and Weese (1993) and Cho (2009). As the arc length method is more susceptible to the characteristics of the overlap of the two isotherms, we considered only the method of Honerkamp and Weese (1993). A modification of this method was proposed to reduce the order of the polynomial and minimise extrapolating data, as shown in Algorithm 1 and graphically illustrated in Figure 13.

Step 1: Determine the shift factor ($a_T^{n+1,n}$) applied for $(n+1)^{th}$ isotherm according to n^{th} isotherm

- Fitting each data set by a (quadratic) polynomial, noted $f_n[\log(t)]$; $f_{n+1}[\log(t)]$
- Given an initial value of $a_T^{n+1,n}$, calculating the value of $f_{n+1}[\log(t/a_T^{n+1,n})]$
- Determining $a_T^{n+1,n}$ by minimising:

$$\Delta f^2 = \sum_{j=j_1}^{j_2} \{f_n(t_j) - f_{n+1}[\log(t_j) - \log(a_T^{n+1,n})]\}^2,$$

wherein $f_n[\log(t_{j_1})]$ and $f_n[\log(t_{j_2})]$ define the potential overlapping region of the two isotherms

Step 2: Determining the shift factor applied for an arbitrary isotherm according to the isotherm at T_{ref}

- Let the k^{th} isotherm be the reference isotherm, the shift factor applied for an arbitrary n^{th} isotherm according to the isotherm at T_{ref} :

$$a_T^{n,k} = \prod_{i=k}^n a_T^{i,i+1} \text{ if } n \geq k; a_T^{n,k} = \prod_{i=n}^k a_T^{i,i+1} \text{ if } n < k.$$

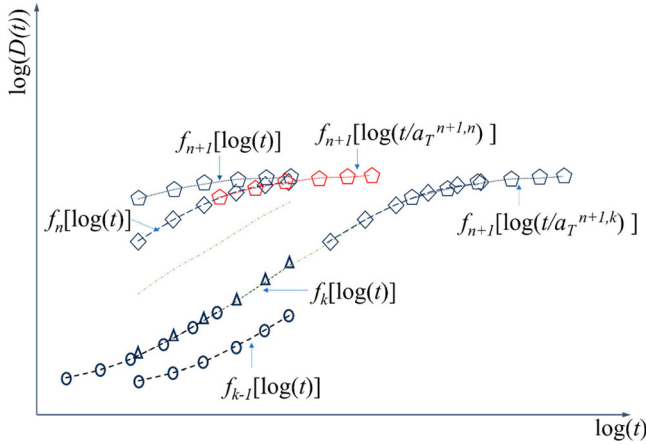


Figure 13. Illustration of the proposed shifting method based on Honerkamp and Weese (1993)'s shifting method.

3.2.2. Master curves of CC, total, and VP strain

Figures 14–16 plot the master curves of CC and the corresponding shift factors of AC, PMB, and HiMA mixture, respectively. It can be seen that the results obtained from the three samples agree well with each other. The RDs between all master curves of CC and their average curve are lower than 11.80% for AC mixture, 6.52% for PMB mixture, and 6.27% for HiMA mixture. Relating to shift factors, the RDs are lower than 5.96% (see Table 3). It is worth mentioning that the master curves of CC and the shift factors

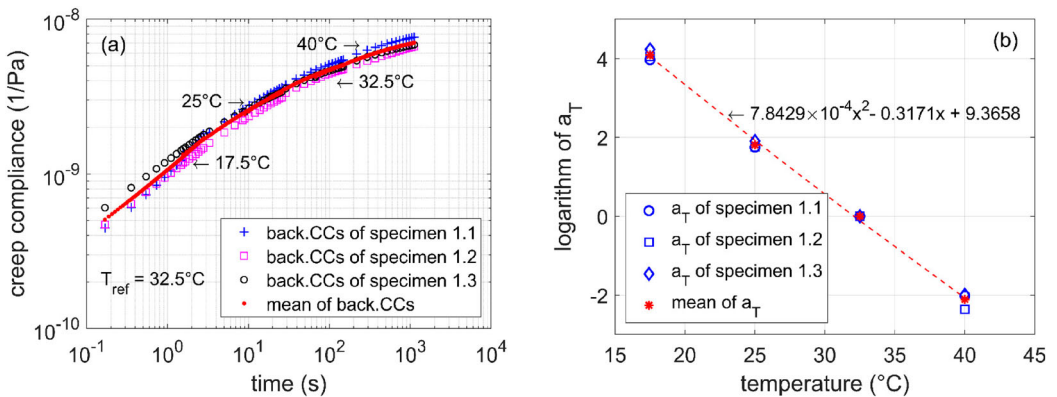


Figure 14. Master curves of all back-calculated CCs of AC mixtures (a) and their corresponding shift factors (b).

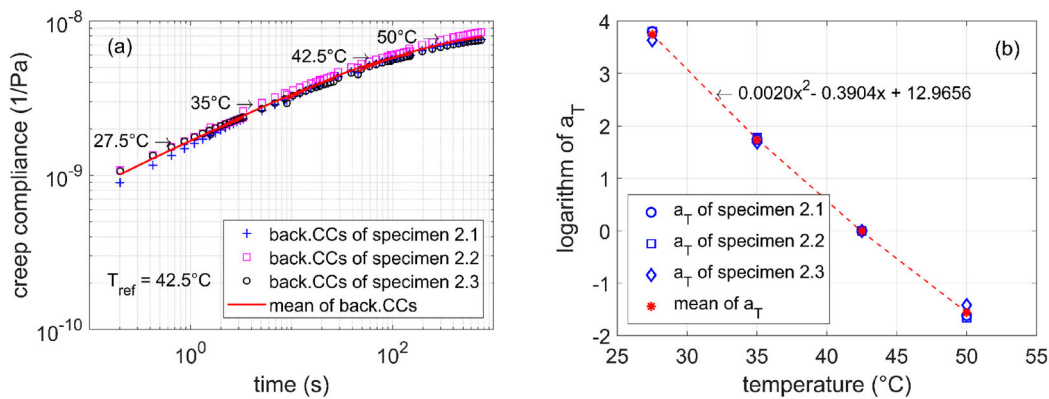


Figure 15. Master curves of all back-calculated CCs of PMB mixtures (a) and their corresponding shift factors (b).

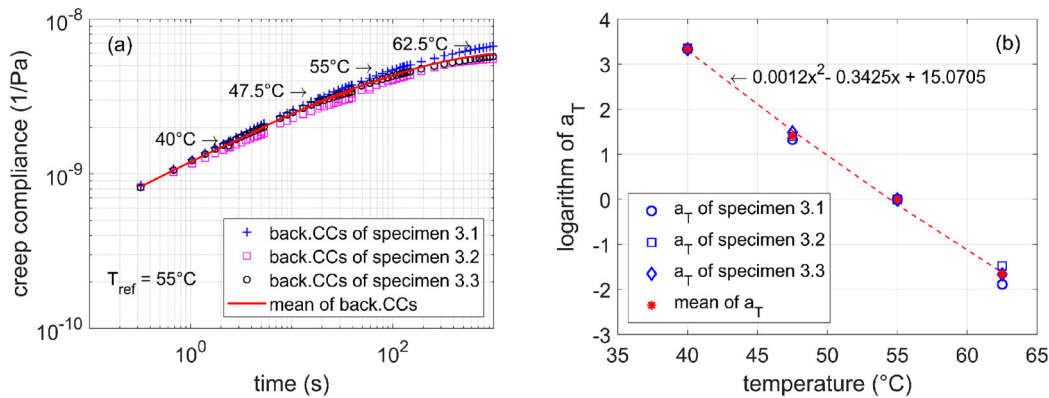


Figure 16. Master curves of all back-calculated CCs of HiMA mixtures (a) and their corresponding shift factors (b).

Table 3. Inter-sample discrepancy of CCs and VE shift factors.

Mixture	Sample code	RDs of CCs	RDs of a_T
AC	1.1	0.0903	0.0324
	1.2	0.0940	0.0522
	1.3	0.1180	0.0416
PMB	2.1	0.0652	0.0180
	2.2	0.0432	0.0274
	2.3	0.0323	0.0428
HiMA	3.1	0.0497	0.0596
	3.2	0.0627	0.0471
	3.3	0.0147	0.0204

of all mixtures were constructed with different testing specimens and stress levels. Despite that, the maximal value of the RDs is small, $\leq 11.80\%$ for CCs, and $\leq 5.96\%$ for shift factors. Owing to the little discrepancy of the air void in specimens as reported in Section 2.1., this good level of repeatability proves the linearity of VE properties with the applied stress levels.

The obtained total and VP strains of AC, PMB and HiMA mixtures were also horizontally shifted using the algorithm proposed in Section 3.2.1 and are displayed in Figures 17–22(a). The shift factors used to construct the master curves of total and VP strain, respectively noted as $a_{T,T}$, $a_{T,VP}$, are also shown in Figures 17–22(b). The overlaps of VP strain isotherms in Figures 20–22(a) result from the fact that the LVDTs were all reset to zero before conducting a new SSCR test as above-mentioned in the beginning

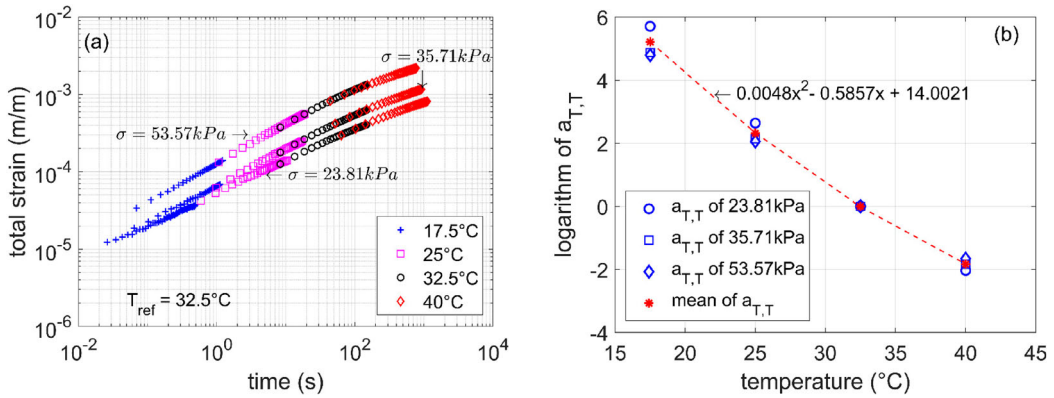


Figure 17. Master curves of total strain of AC mixtures (a) and their corresponding shift factors (b).

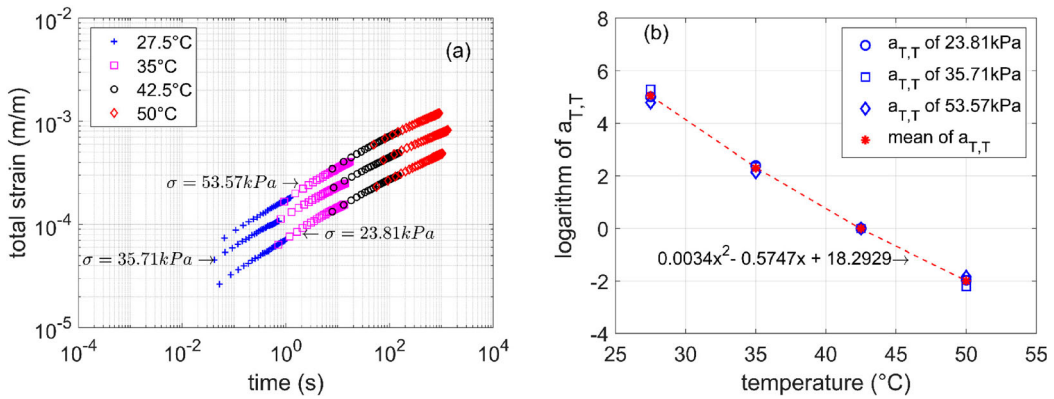


Figure 18. Master curves of total strain of PMB mixtures (a) and their corresponding shift factors (b).

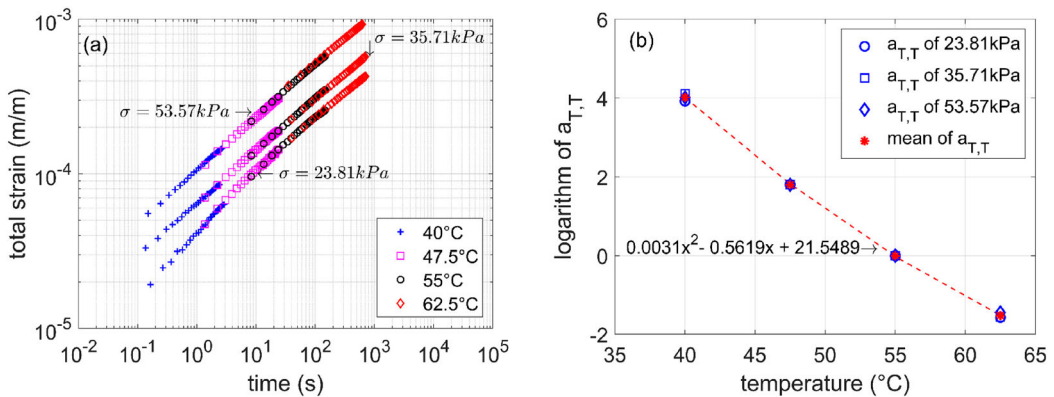


Figure 19. Master curves of total strain of HiMA mixtures (a) and their corresponding shift factors (b).

of Section 3. It can be observed that the set of all the shifted curves of total and VP strain seems to form a single and relatively smooth curve. In addition, the curves are monotonous, which ensures an injective relationship between time and total or VP strain. Thus, the curve formed by all shifted curves of total and VP strain can be considered the master curve of total and VP strain at the reference mean temperature.

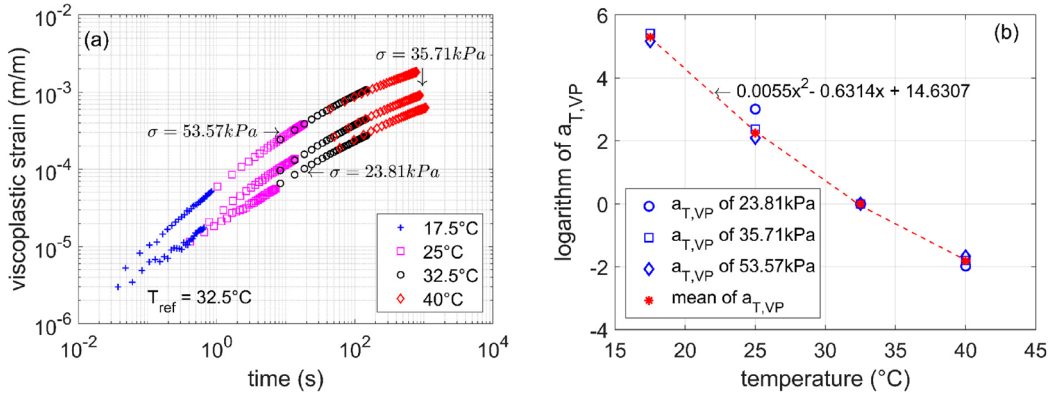


Figure 20. Master curves of VP strain of AC mixtures (a) and their corresponding shift factors (b).

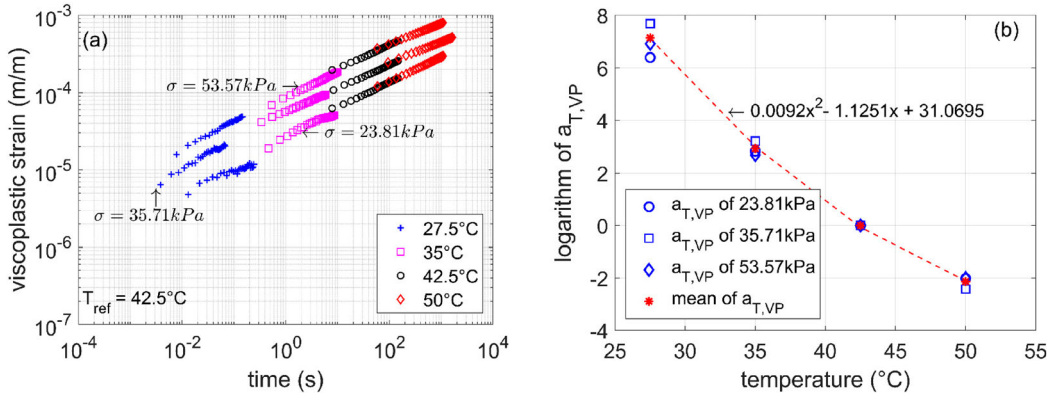


Figure 21. Master curves of VP strain of PMB mixtures (a) and their corresponding shift factors (b).

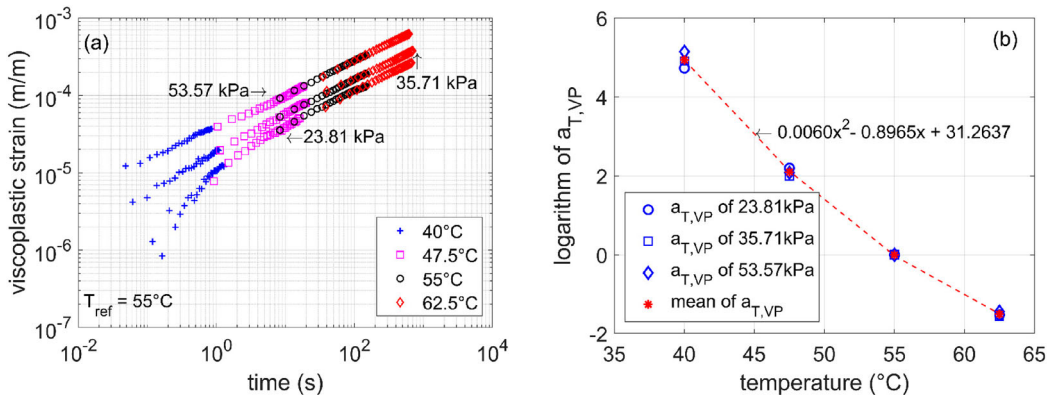


Figure 22. Master curves of VP strain of HiMA mixtures (a) and their corresponding shift factors (b).

Furthermore, the functions of all shift factors ($a_{T,T}$ and $a_{T,VP}$) are also monotonous, which ensures a unique relationship between temperature and $a_{T,T}$ or $a_{T,VP}$. Therefore, TTSP can be considered valid for both total and VP strain of the mixtures investigated.

With regards to the discrepancy caused by inter-specimen heterogeneity and different stress levels on the obtained $a_{T,T}$ and $a_{T,VP}$, the largest value of RDs of all total and VP shift factors and their average values is respectively 10.42% and 15.09% for AC mixture, 5.93% and 9.43% for PMB mixture, and 2.30% and 4.23% for HiMA mixture. The values are small and well-accepted in pavement engineering; thus, it can be concluded that the total and VP shift factors do not depend on the applied stress level.

3.2.3. Comparison of total, VE, and VP shift factors

In order to compare the shift factors used to construct the master curves of total strain, CC, and VP strain, the values of $a_{T,T}$, a_T , and $a_{T,VP}$ are once again plotted in the same figure (Figure 23). Factors causing biases in the values of $a_{T,T}$, $a_{T,VP}$ such as the inaccurate application of the pure creep loading history and effects of the cumulative permanent strain on successive SSCR testing were carefully investigated to quantitatively estimate the reliable intervals of the shift factors. For instance, the error bars attached to the graphs of $a_{T,T}$ and $a_{T,VP}$ denote their possible lower and upper bounds while those displayed in the plot of a_T are just the SD of the set of three values of a_T obtained from three SSCR tests. The detailed discussion on the bias estimation for $a_{T,T}$ and $a_{T,VP}$ will be presented in Section 3.3.

As can be seen in Figure 23, $a_{T,T}$ and $a_{T,VP}$ corresponding to elevated temperatures (as compared to the performance of the studied mixture, e.g. $\geq 32.5^\circ\text{C}$ for AC mixture, $\geq 42.5^\circ\text{C}$ for PMB mixture, and $\geq 55^\circ\text{C}$ for HiMA mixture) seem to have similar values with VE shift factors. In fact, at these temperatures, the mean RD between $a_{T,T}$ and a_T is 13.76% for AC mixture, 26.98% for PMB mixture, and

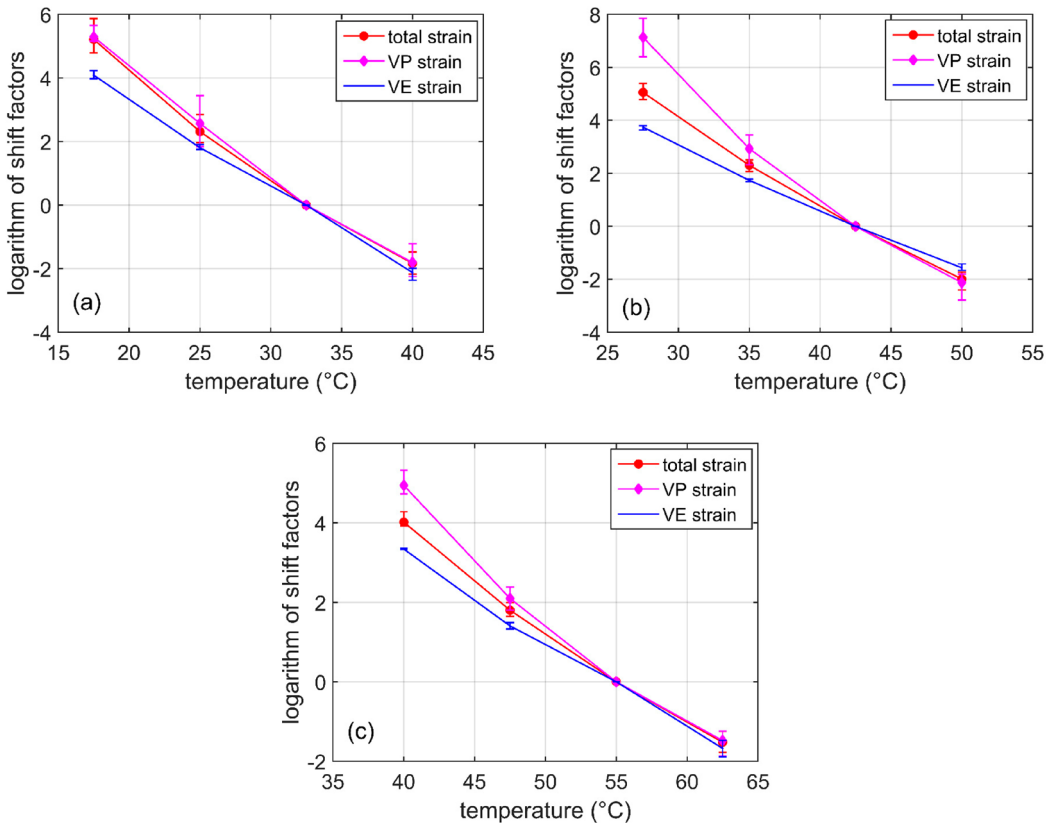


Figure 23. Shift factors of total, VE, and VP strain obtained with (a) AC, (b) PMB, and (c) HiMA samples. The error bars in the graphs of $a_{T,T}$ and $a_{T,VP}$ indicate the reliable interval of shift factors at a testing temperature considering biases caused by accumulated permanent contraction strains while the error bars in the graphs of a_T show the variation of a_T obtained from SSCR tests with different stress levels.

Table 4. Relative differences between mean values of shift factors.

RD Mixture	AC	PMB	HiMA
$a_{T,T}$ vs. a_T (%)	25.639	33.744	20.034
$a_{T,VP}$ vs. a_T (%)	29.528	82.440	44.007
$a_{T,VP}$ vs. $a_{T,T}$ (%)	4.487	36.907	20.876

Table 5. Potential lowest RDs between shift factors.

RD Mixture	AC	PMB	HiMA
$a_{T,T}$ vs. a_T (%)	11.419	23.367	14.932
$a_{T,VP}$ vs. a_T (%)	18.947	60.156	35.366

7.04% for HiMA mixture while the mean RD between $a_{T,VP}$ and a_T is 14.95% for AC mixture, 35.71% for PMB mixture, and 11.73% for HiMA mixture. These rather small gaps of total and VP shift factors of AC and HiMA mixtures estimated at elevated temperatures may explain the suggestion of many authors to extend the validity of VE shift factors into the VP domain.

However, in considering the full range of the investigating temperatures, the differences between $a_{T,T}$ and a_T , and between $a_{T,VP}$ and a_T become significant. As reported in Table 4, the RDs between the mean values of $a_{T,T}$ and a_T may reach 33.74% for the case of PMB mixture while the gaps between the mean values of $a_{T,VP}$ and a_T are even higher with the RD exceeding 82.44% for the same mixture. Even though the discrepancies of all shift factors are considered, the gaps between $a_{T,T}$ and a_T , and between $a_{T,VP}$ and a_T of all mixtures are still clearly recognisable. In fact, the potential shortest distances between $a_{T,T}$ and a_T of AC, PMB, and HiMA mixture are respectively 11.42%, 23.37%, and 14.93% (see Table 5), while the possible minimal gaps between $a_{T,VP}$ and a_T are much higher, 18.95% for AC mixture, 60.16% for PMB mixture, and 35.37% for HiMA mixture. This indicates the obvious independence of the total and VP shift factors on the VE shift factors as already pointed out in (Cao & Kim, 2022).

An interesting point to be mentioned is that $a_{T,T}$ and $a_{T,VP}$ of AC mixture, or the unmodified one, seem to share the same values at all testing temperatures with the RD between them being only 4.49%. However, this property does not necessarily lead to a dependent relationship between $a_{T,T}$ or $a_{T,VP}$ with the VE shift factor. Apart from this case, the graphs of $a_{T,T}$ and $a_{T,VP}$ show obvious gaps between them, proving the necessity of using independent values of the total, VP, and VE shift factors in the characterisation and modelling of the full-range stress–strain behaviour of asphaltic materials.

3.2.4. Comparison of the resistances to PD of mixtures

With the constructed master curves of VP strain, the resistance to PD of the three mixtures can be compared by plotting the master curves of VP strain caused by the same stress levels applied and at the same reference temperature (40°C) (see Figure 24). Overall, HiMA mixture has the highest resistance to permanent deformation, followed by PMB mixture, and AC mixture is the least rut resistant. For instance, in order for the VP strain to attain a value of 10^{-4} (m/m) with the applied stress equal to 23.81 kPa, the loading time needed was approximately 7922s for HiMA mixture, 101.6s for PMB mixture, and 2.57s for AC mixture (see Figure 24a). With a fixed value of loading time, e.g. at 100s, the VP strains of HiMA mixture, PMB mixture, and AC mixture were respectively 3.28×10^{-5} , 3.28×10^{-4} , and 1.61×10^{-3} (m/m) when the stress level was 53.57 kPa. The differences are large enough to effectively rank the resistance to PD of all mixtures.

It should be mentioned that the VP strains of PMB mixtures obtained with the stress levels of 35.71 and 53.57kPa at a loading time lower than 0.12s were larger than those of AC mixtures (see Figure 24b and c). This may suggest that the resistance to PD at an extremely short loading time of PMB mixtures is lower than that of AC mixtures. However, the order of magnitude of the VP strains at that range of loading time was small, approximately 10^{-5} (m/m). Thus, this observation may stem from the errors of measurement devices and needs to be checked in further studies.

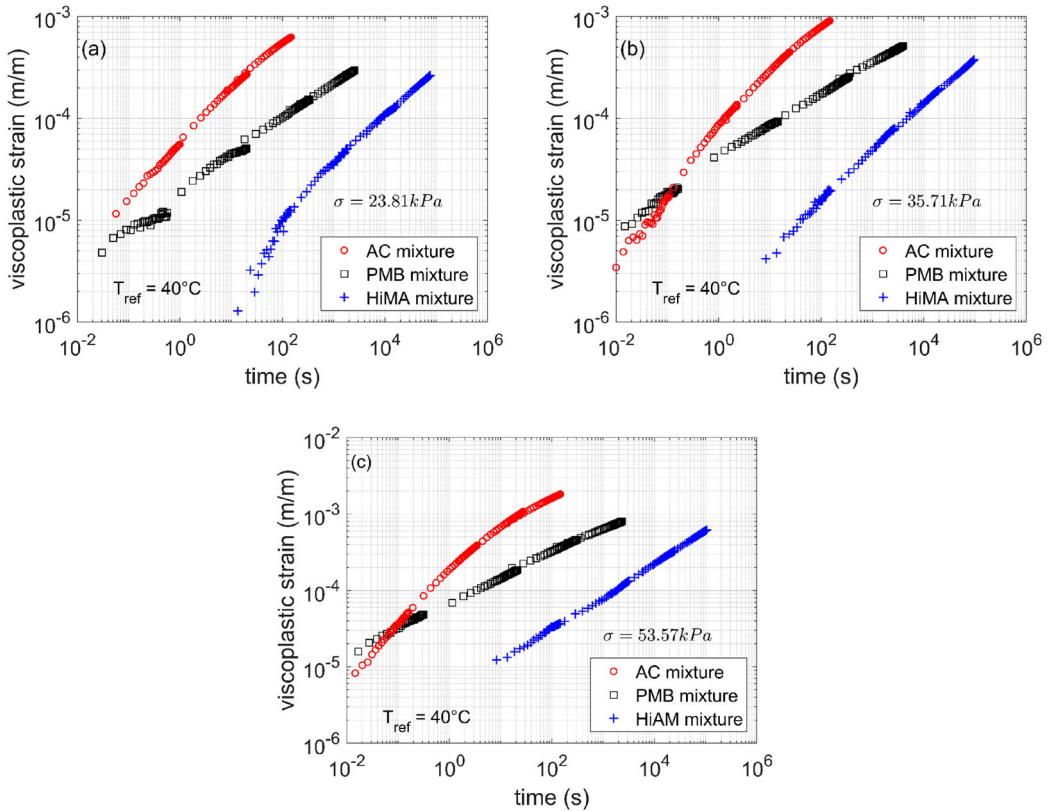


Figure 24. Comparison of the resistance to PD of all mixtures at different stress levels: (a) 23.81 kPa, (b) 35.71 kPa, and (c) 53.57 kPa.

3.3. Factors causing biases in the obtained results

3.3.1. Effects of non-instantaneous application of the stress

With a hydraulic servo actuator, the instantaneous stress cannot be exactly applied. In fact, the true creep loading history was approximated by applying a ramp loading function during a short period, called loading ramp time, prior to the application of a constant loading function (see Figure 25a). This means that the response strains measured including VE and VP components are significantly lower than the true values at the beginning of the test. Owing to the viscous properties of the materials studied, the effects of the non-instantaneous application of loading rapidly attenuate, and the measured strain approaches its true value as the creep time exceeds some moment, noted as t_c . Therefore, the response strains measured at $t < t_c$ are less reliable and need to be cut off from the results.

According to the results reported in Appendix A1., in order to ensure the tolerance of 2% for the obtained results, the cut-off times for VE component range from 6.46 to 11.83 times t_0 while the cut-off times for VP strain are slightly higher, varying from 9.51 to 15.15 times t_0 . Therefore, with the loading ramp time being set to 0.5s in this study, the measured data before 7.5s were cut off to mitigate the effects of the non-instantaneous application of the stress on the obtained results.

3.3.2. Replication of VP strains and effects of accumulated permanent contraction strain on the measured total and VP strains, and their companion shift factors

In this study, the determination of VP strains at four temperatures in ascending order caused by one stress level was performed on one single specimen instead of four different virgin specimens. Before running an SSCR test, the specimens were left free of stress for approximately 20 h at room temperature

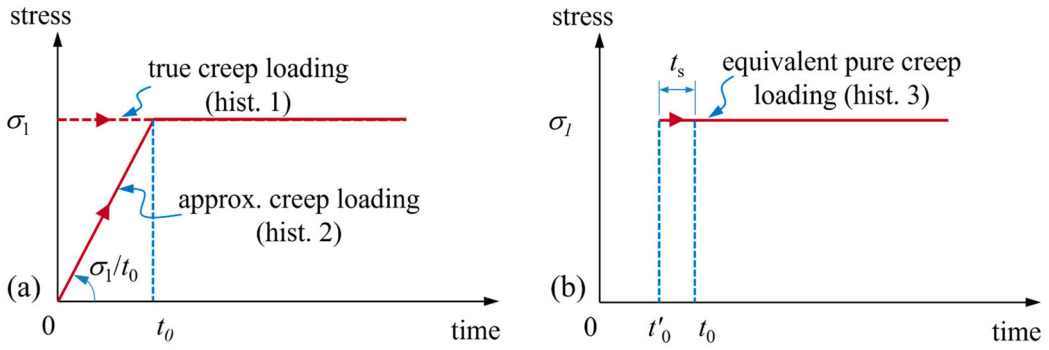


Figure 25. Illustration of the applied creep loading history at the beginning of an SSCR test. (a) ramp stress history created by the hydraulic servo actuator used. (b) conversion of ramp stress history into a pure creep stress history.

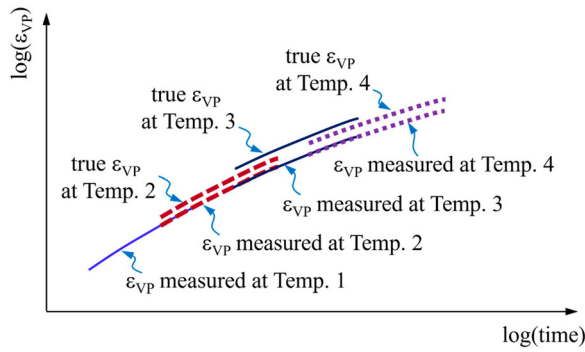


Figure 26. Illustration of the effect of cumulative permanent contraction strain on the measured strains.

for the material to restore as much as possible to its initial conditions, and the LVDTs were reset to zero so that the measured response strain of the specimen in an SSCR test can be considered independent on those obtained in other SSCR tests. With this approach, the effect of inter-specimen heterogeneity is eliminated, and the experimental cost is significantly reduced. However, the VP strain accumulated during preceding SSCR tests certainly affects the development of VP strain in subsequent ones as already pointed out in (Tran et al., 2023). In fact, after performing an SSCR test, the permanent contraction strain inside the specimen increases by an amount equal to the VP strain developed in that SSCR test, which enhances the aggregate interlock and makes the material harden. In other words, the material of the specimen is not virgin before running SSCR tests except the test at the lowest temperature, which makes the measured total and VP strains lower than their true values except the one obtained at the lowest temperature as illustrated in Figure 26. This phenomenon certainly causes biases in the construction of the master curves of total and VP strain.

Therefore, tests were consecutively performed from the lowest to the highest temperatures without alternating repeatability tests to minimise the influence of the cumulative permanent strain on the obtained results. Instead, the replication of the total and VP strains under the same testing conditions was assessed after all SSCR tests had been finished. Three replicate tests for three mixtures were carried out on specimens 1.3, 2.1, and 3.3 to assess the extent of reduction in the total and VP strain induced by the non-virgin conditions of the material before conducting SSCR tests. Since VP strains obtained at the lowest temperatures were not affected by the accumulated permanent contraction strain, no replicate test was conducted at that temperature. The results of the total and VP strains gained in replication tests are all presented in Figures 27–29, which clearly show a considerable reduction in their values as

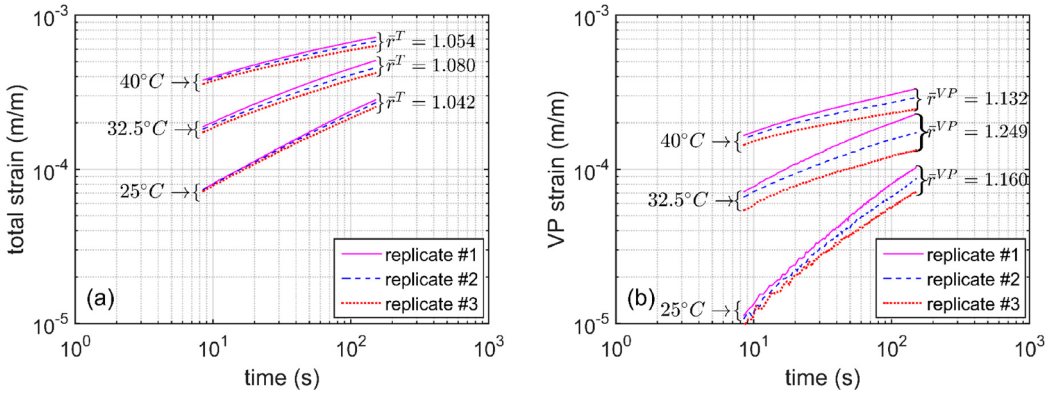


Figure 27. Results of replication test of AC mixture at 25, 32.5, and 40°C. (a) Total strains, (b) VP strains.

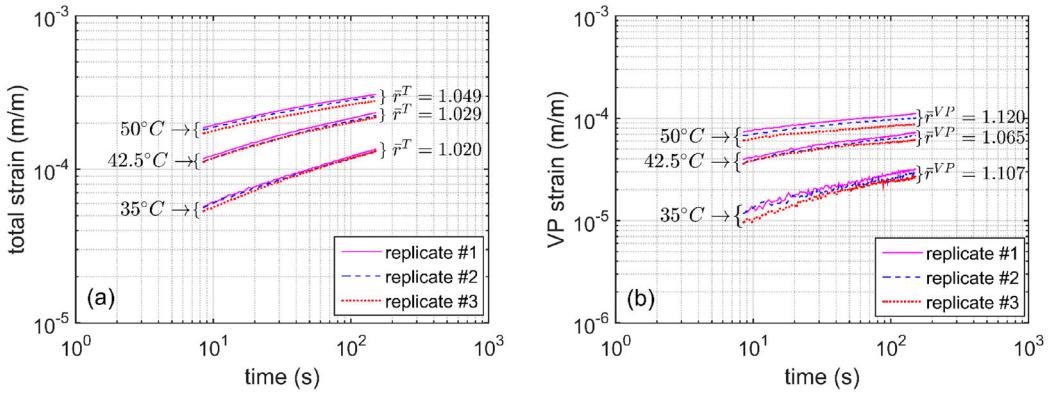


Figure 28. Results of replication test of PMB mixture at 35, 42.5, and 50°C. (a) Total strains, (b) VP strains.

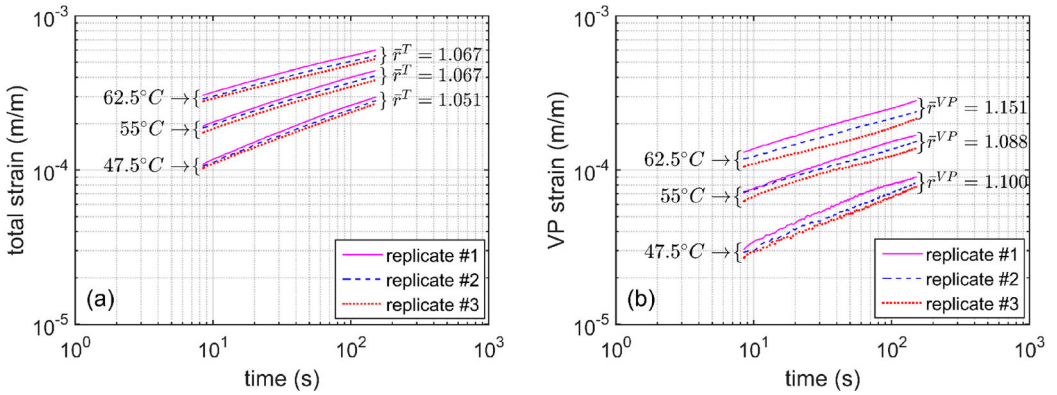


Figure 29. Results of replication test of HiMA mixture at 47.5, 55, and 62.5°C. (a) Total strains, (b) VP strains.

compared with initial SSCR tests due to the extremely high amount of the permanent strain already accumulated in the specimens.

It can be seen in Figure 27–29 that the total and VP strains developed in a subsequent SSCR test are lower than those in preceding SSCR tests due to mechanical hardening effects as previously mentioned. Let r_i^T and r_i^{VP} be the reduction ratios of the total and VP strains at a testing temperature defined

Table 6. Reduction ratios of VP and total strains.

Mixture	AC			PMB			HiMA		
Temp. (°C)	25	32.5	40	35	42.5	50	47.5	55	62.5
$\bar{r}^T$	1.042	1.08	1.054	1.020	1.029	1.049	1.051	1.067	1.067
$\bar{r}^{VP}$	1.116	1.249	1.132	1.107	1.065	1.120	1.100	1.088	1.151

as the average ratios of the total and VP strains measured in test replicate i to those measured in test replicate $(i + 1)$. As a result, the average reduction ratios of the total and VP strains at that temperature after one cycle of CR test were calculated as $\bar{r}^T = \sum r_i^T / 2$, $\bar{r}^{VP} = \sum r_i^{VP} / 2$, and are reported in Table 6. The data show that the values of $\bar{r}^T$ and $\bar{r}^{VP}$ are relatively small, respectively lower than 1.08 and 1.25, proving the relatively good level of repeatability of the measured total and VP strains.

Let us now respectively define R^T and R^{VP} as the average ratio of the true total and VP strains to the measured ones at a testing temperature. Since the values of VP strain at the end of an SSCR test are almost greater than the amount of the cumulative VP strains gained in all previous SSCR tests (see Figure 20–22), one can assume that $R^T \leq \bar{r}^T$ and $R^{VP} \leq \bar{r}^{VP}$. With this assumption in hand, the reliable intervals of shift factors for the total strains at a testing temperature T_j can be estimated:

$$\log(a_{T,T}) - \frac{\log(R^T(T_j))}{m_s} \leq \log(a_{T,T}) \leq \log(a_{T,T}) + \frac{\log(R^T(T_{ref}))}{m_s}, \quad \text{if } T_j > T_{ref}, \quad (13)$$

$$\log(a_{T,T}) - \frac{\log(R^T(T_{ref}))}{m_s} \leq \log(a_{T,T}) \leq \log(a_{T,T}) + \frac{\log(R^T(T_j))}{m_s}, \quad \text{if } T_j < T_{ref}, \quad (14)$$

where m_s is the average slope coefficient of $\log(\varepsilon_T)$ versus $\log(t)$ in the overlapping region of the two isotherms. The equations for VP shift factors are similar to Equations (13)–(14) and will not be presented here for the sake of brevity. The lower and upper bounds of $a_{T,T}$ and $a_{T,VP}$ were calculated and are all presented in Figure 23. The largest discrepancies of the total and VP shift factors were respectively $\pm 23.17\%$ and $\pm 34.05\%$ (of the mean values) for AC mixture, $\pm 20.07\%$ and $\pm 31.04\%$ for PMB mixture, and $\pm 18.37\%$ and $\pm 26.32\%$ for HiMA mixture.

3.4. Agreement between the results of LVE characteristics obtained in the time and frequency domain

The agreement between the results of LVE characteristics of asphalt mixtures obtained in the time and frequency domain needs to be verified to ensure that VP strain was accurately determined. After conducting SSCR tests, dynamic modulus tests were also performed to determine the LVE behaviour of the mixtures in the frequency domain. Since tests in the frequency domain are well-known and widely used in the community, tests were carried out using only one sample per mixture (samples 1.2, 2.3, and 3.1). The master curves of dynamic modulus and phase angle were constructed using the horizontal shifting technique presented in Algorithm 1 and predicted using the Huet-Sayegh model. The complex modulus function was then converted into the CC function using the algorithm proposed in (Phan et al., 2022) to compare with the CCs back-calculated from SSCR tests. All the obtained results will be presented in the next subsections.

3.4.1. Dynamic modulus test

The test set-up was the same as that used for SSCR tests. A haversine shape compressive stress was applied to the sample and the response strain was measured. The applied stress was carefully chosen such that the strain amplitude did not exceed $75\mu\varepsilon$ as suggested in (Nguyen et al., 2021b) for the behaviour of the mixture to remain in the LVE domain. Testing temperatures were $-10, 0, 10, 20, 30, 40$, and 50°C for all mixtures and an extra temperature (60°C) was investigated for HiMA mixture.

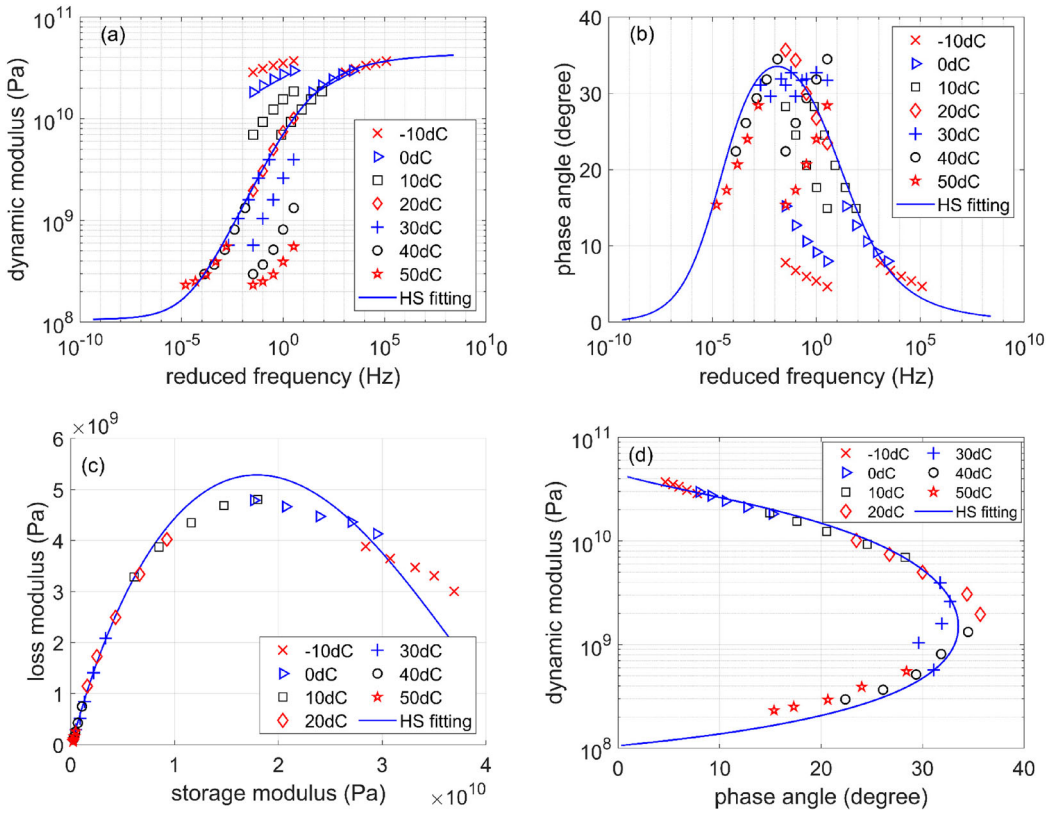


Figure 30. Results of LVE characterisation of AC mixture. (a) Dynamic moduli (b) phase angles, (c) Cole – Cole diagram, and (d) Black diagram.

At each testing temperature, five frequencies (0.033, 0.1, 0.33, 1, and 3.33 Hz) were considered. The raw data were post-processed using the algorithm presented in (AASHTO T342-11, 2011) to calculate the dynamic moduli and phase angles. The reference temperature equal to $T_{ref} = 20^\circ\text{C}$ was chosen for all mixtures.

Figures 30–32a and b respectively display the master curves of dynamic modulus and phase angle of AC, PMB, and HiMA mixtures and the corresponding shift factors are presented in Figure 33. Meanwhile, Figures 30–32c and d show the Cole–Cole and Black diagrams of all mixtures. It can be seen that all components of the complex modulus formed a single and relatively smooth curve; thus, TTSP is applicable for all mixtures in the LVE domain.

In order to predict the LVE characteristics of all mixtures, the HS parameters were determined such that the errors between the predicted and experimental dynamic moduli were minimal, except E_0 , k , h . As suggested in many studies, parameters k , h were chosen from Cole–Cole diagrams to reduce the number of variables in the optimisation process. Based on our observations, E_0 has little impact on the errors between the experimental and predicted dynamic moduli yet it affects much the results of time–frequency interconversion.

Therefore, in order to ensure the objectivity in determining E_0 , its values were measured based on static modulus tests conducted at a temperature higher than the highest temperature used in dynamic modulus (5–10°C higher). Loading and unloading time in static modulus tests were respectively 300s and 1500s. The static modulus E_0 can be calculated by $E_0 = \sigma / \Delta\varepsilon$, where σ is the stress level applied to the sample, $\Delta\varepsilon$ is the recovered strain during the unloading period. The Huet-Sayegh parameters are all presented in Table 7.

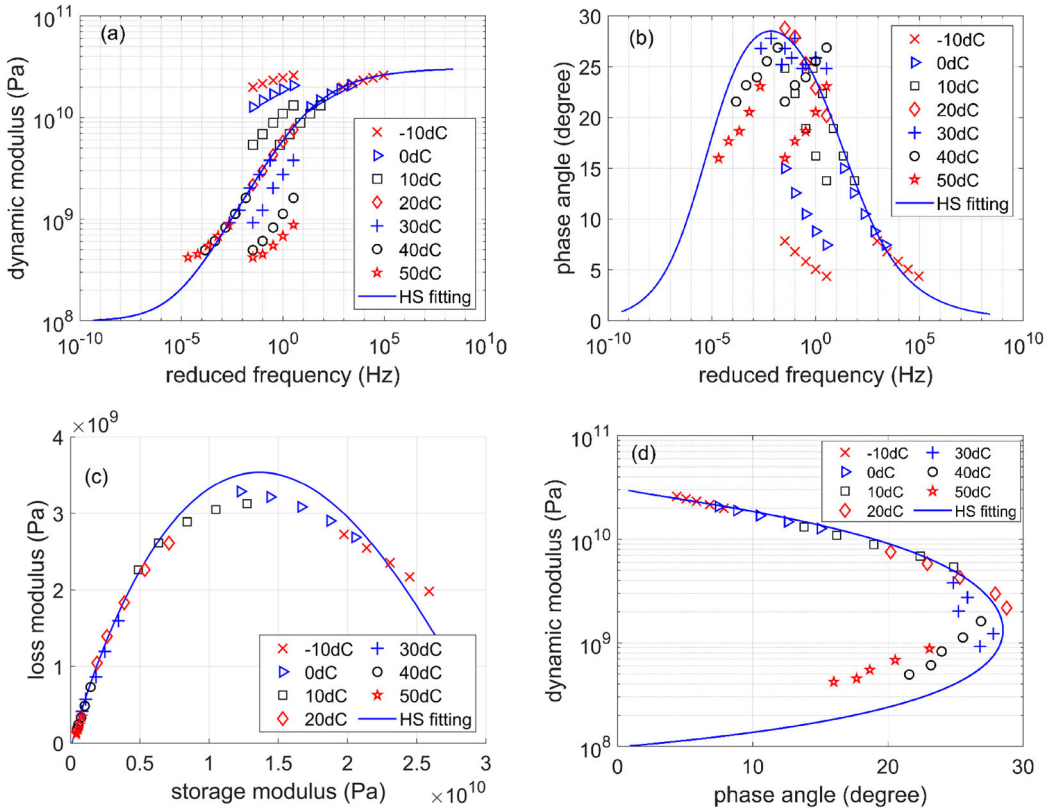


Figure 31. Results of LVE characterisation of PMB mixture. (a) Dynamic moduli (b) phase angles, (c) Cole – Cole diagram, and (d) Black diagram.

With the determined Huet-Sayegh parameters, the predicted LVE characteristics were computed and are presented in Figures 30–32. As can be seen in the figures, Huet-Sayegh model represents very well the LVE behaviour of all mixtures—namely, the relative errors between the predicted and experimental dynamic modulus are lower than 2.94% while the relative errors between the predicted and experimental phase angles are lower than 12.12% (see Table 8). It is clear that the relative errors of the predicted phase angles of all mixtures are higher than those of the predicted dynamic moduli. This may stem from the higher errors of phase angle measurement as already discussed in (Phan et al., 2022; Rowe et al., 2009; Zhang et al., 2020).

3.4.2. Comparison of VE shift factors in the frequency and time domain

The shift factors used to construct the master curve of dynamic modulus were determined using the algorithm presented in Section 3.2.1. In order to compare the shift factors in the frequency and time domain, their values were converted into those at one common temperature at which both shift factors in the frequency and time domain were experimentally determined to avoid interpolation. As a result, the new reference temperature of AC and HiMA mixtures was 40°C while that of PMB mixture was 50°C (see Figure 34). It can be shown that the shift factors in frequency and time domain agree relatively well with each other as already discussed in Nguyen et al. (2021b). In fact, the shift factors in the frequency domain of AC, PMB, and HiMA differ respectively by 17.94%, 4.75%, and 5.97% from those in the time domain.

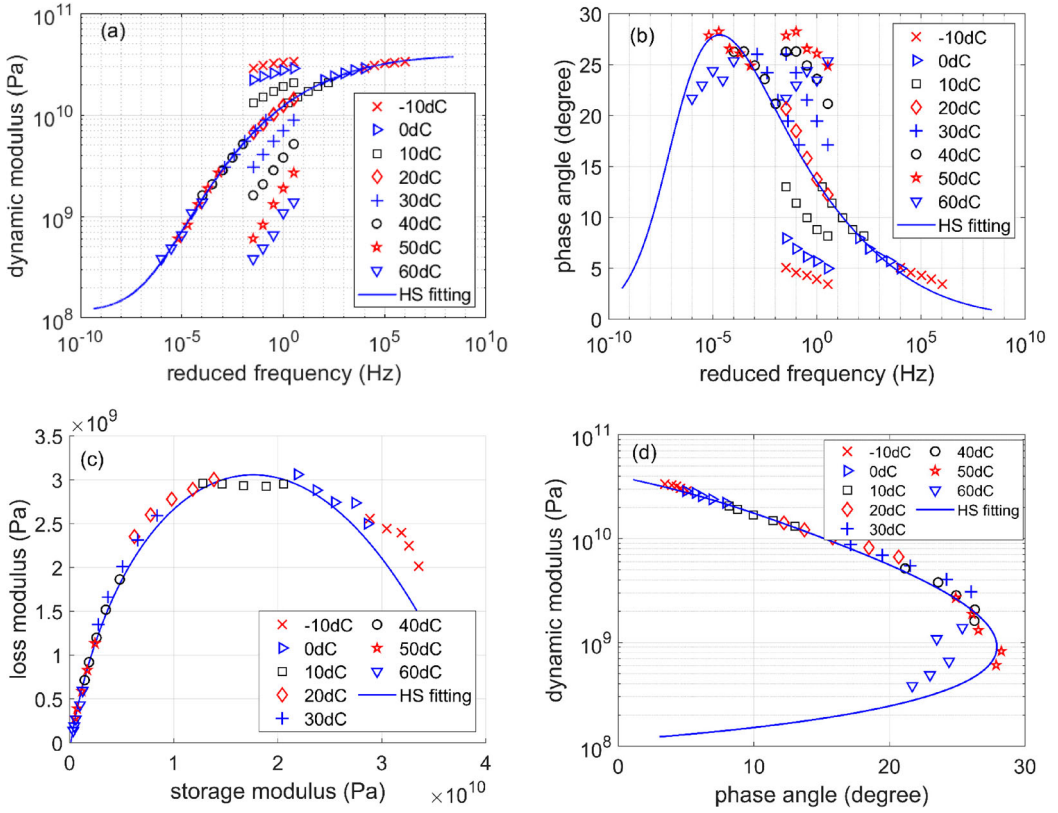


Figure 32. Results of LVE characterisation of HiMA mixture. (a) Dynamic moduli (b) phase angles, (c) Cole – Cole diagram, and (d) Black diagram.

3.4.3. Comparison of creep compliances

A variety of methods have been proposed in the literature for time–frequency interconversion of LVE properties, which can be classified into two main groups: numerical and analytical methods. For instance, Baumgaertel and Winter (1989), Mead (1994), Mun et al. (2007), and Zhang et al. (2018) fit the DM data by the generalised Maxwell model and then the CC function can be easily deduced. However, it is difficult to properly determine the retardation times due to the nonlinearity of the problem. In addition, the waviness of the predicted results was often observed when fitting the noisy experimental data. Regarding analytical approaches, the method proposed by Schapery and Park (1999) was widely used for its simplicity and explicit formulas. Since the method used some analytical approximations in the formulation, the errors of the interconverted results may reach 5% (Emri et al., 2005; Nguyen et al., 2021b).

The objective of this section is to verify the agreement between LVE properties in the frequency and time domains; thus, a much more accurate method for time–frequency interconversion must be applied. For that purpose, the method proposed by Phan et al. (2022) was used to compute the CCs from the DM data. In order to facilitate the reading of this paper, this method is summarised as follows. First, the storage compliance was calculated by:

$$D'(\omega_i) = \cos(\varphi(\omega_i)) / |E^*(i\omega_i)|, \quad (15)$$

where $D'(\omega_i)$ is the storage compliance, $\varphi(\omega_i)$ is the phase angle, and $|E^*(i\omega_i)|$ is the dynamic modulus. Next, the storage compliance was decomposed into the sum of exponential functions (SEFs) using the

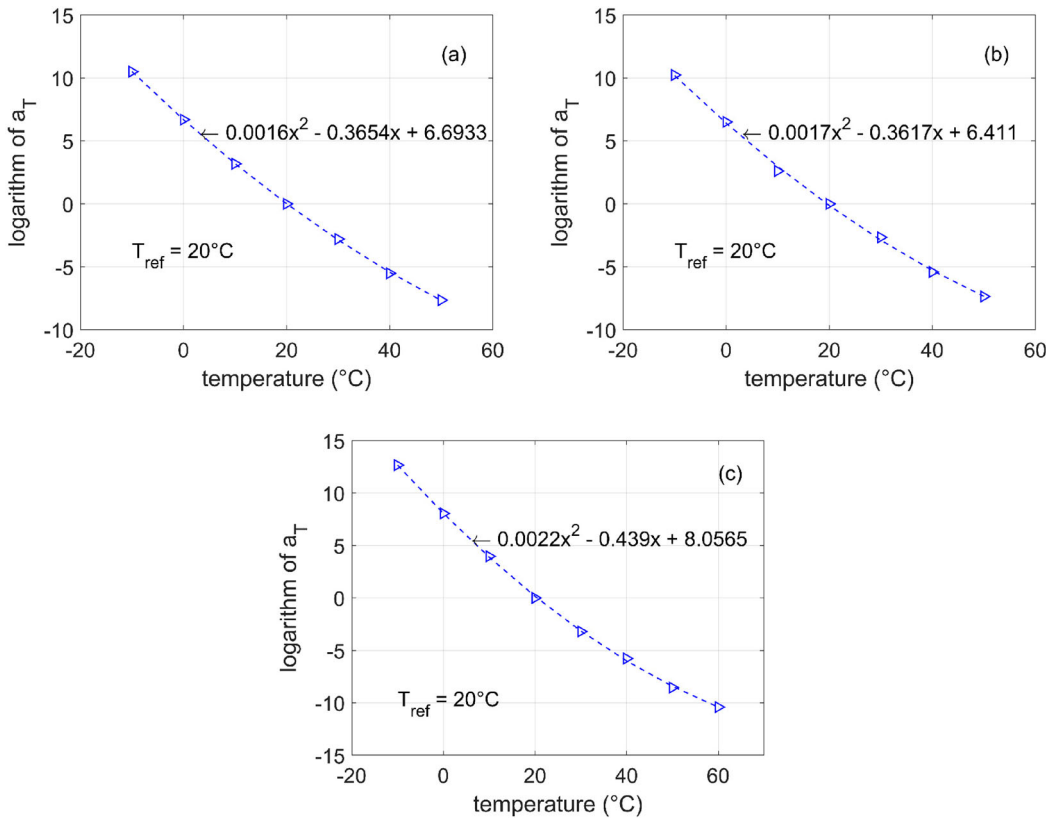


Figure 33. VE shift factors of (a) AC mixture, (b) PMB mixture, and (c) HiMA mixture.

Table 7. Huet-Sayegh parameters of all studied mixtures.

Mixture	E_0 (MPa)	E_∞ (MPa)	k	h	δ	τ (s)	T_{ref} ($^\circ\text{C}$)
AC	105.923	44232.438	0.170	0.435	0.770	0.005	20
PMB	98.834	31107.174	0.170	0.375	0.611	0.006	20
HiMA	115.355	39509.649	0.180	0.430	3.121	1.925	20

Table 8. The relative errors (RE) between the experimental and predicted master curve of dynamic modulus and phase angle.

Mixture	$\text{RE}^{[E^*]} \text{ (%)}$	$\text{RE}^\varphi \text{ (%)}$
AC	2.937	9.714
PMB	2.866	12.117
HiMA	2.352	7.602

modified Dombi algorithm presented in (Nguyen et al., 2022):

$$D'(\omega_i) \approx D^*(\omega \rightarrow \infty) + \sum_{j=1}^{N_j} D'_j e^{c_j \omega_i} \quad (16)$$

where c_j is real-valued negative decay rates; D'_j is real-valued coefficients; and N_j is the number of exponential functions. Then, the CC function can be obtained using the inverted Fourier

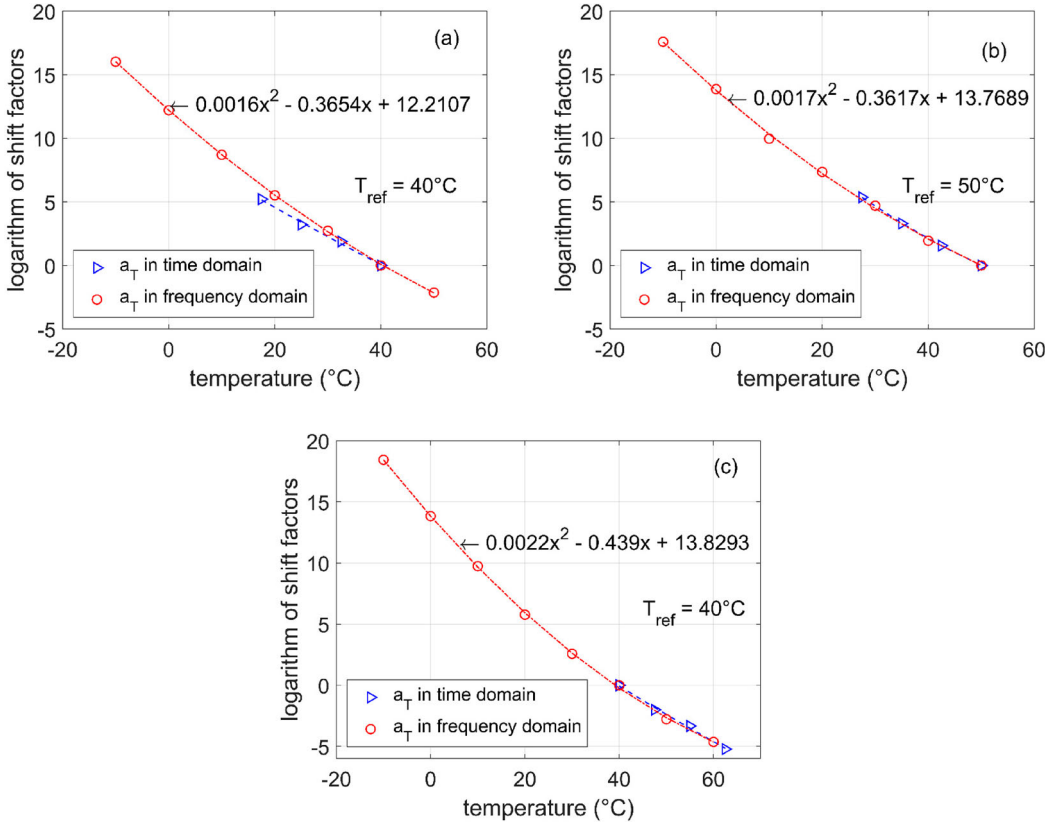


Figure 34. Comparison of VE shift factors in frequency and time domain. (a) AC mixture, (b) PMB mixture, and (c) HiMA mixture.

transform:

$$D(t_i) = D^*(\omega \rightarrow \infty) + \frac{2}{\pi} \sum_{j=1}^{N_j} D'_j \cdot \tan^{-1}(-t_i/c_j) \quad (17)$$

The creep compliances interconverted from DM data and those back-calculated from SSCR tests of samples 1.2, 2.3, and 3.1 are all presented in Figure 35. As can be seen in the figure, all the back-calculated and interconverted CCs are in relatively good accord with each other. The relative differences between them are 4.28% for AC mixture, 8.38% for PMB mixture, and 4.57% for HiMA mixture. This shows that the LVE properties determined from SSCR tests are reliable; therefore, the obtained VP strains are reliable.

3.5. Application of TTSP in VP domain to characterise and predict the permanent deformation behaviour of AC, PMB, and HiMA mixtures

This section is devoted to demonstrating the efficiency of TTSP in characterising and predicting the PD behaviour of all studied mixtures. Thus, the well-known power law model proposed in (Hua, 2000) was adopted to represent the evolution of VP strain in the specimen subjected to a pure creep loading:

$$\dot{\varepsilon}_{VP}(t) = Abt^{b-1}\sigma^m, \quad \varepsilon_{VP}(t) = \int_0^t \dot{\varepsilon}_{VP}(\tau) d\tau = At^b\sigma^m, \quad (18)$$

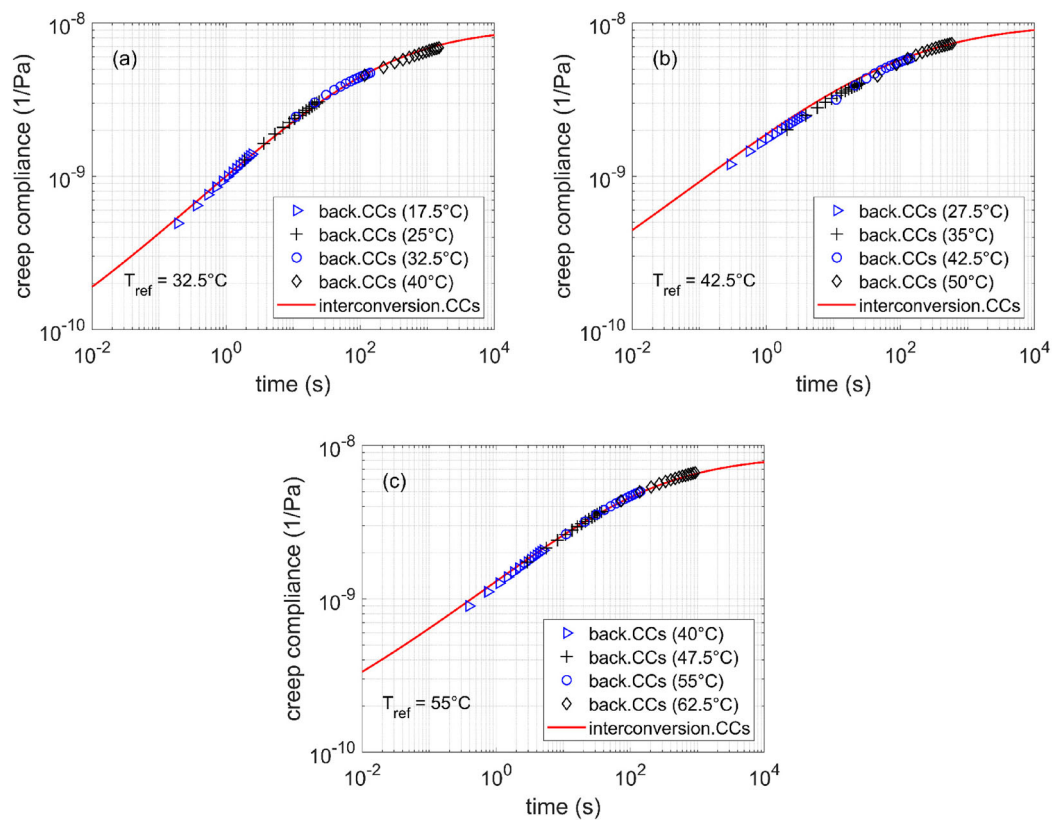


Figure 35. Back calculated CCs versus interconverted CCs. (a) specimen 1.2, (b) specimen 2.3, and (c) specimen 3.1.

Table 9. Best-fit model parameters for predicting VP strains of all mixtures exerted by a pure creep loading (the stress and time are in SI unit).

Mixture	A	b	m
AC	7.296×10^{-12}	0.538	1.459
PMB	6.454×10^{-11}	0.319	1.298
HiMA	2.498×10^{-10}	0.454	1.080

where b is the slope coefficient of the predicted VP strain curve in the double logarithmic scale, m controls the nonlinear effect of the stress level, and A is just a correcting factor. Thus, in order to determine all of the model parameters, at least two master curves of VP strain caused by different stress levels are required. Thus, the master curves of VP strain caused by the lowest and highest stress levels were used to calibrate the model while the master curves of VP strain caused by the remaining stress level were used to assess the accuracy of the prediction. With the aid of the Microsoft Excel Solver toolbox, the sum of squared errors between the predicted and measured VP strains in the double logarithmic scale was minimised to determine the best-fit model parameters as reported in Table 9.

Figures 36–38(a) respectively show the VP strains of AC, PMB, and HiMA mixtures computed with the best-fit parameters at the reference temperature caused by the stress levels of 23.81 and 53.57kPa. It can be observed that the VP model used represents well the evolution of VP strains of PMB and HiMA mixtures because of their straight-line shape in double logarithmic scale plots. Meanwhile, the master curves of VP strain of AC mixture are more complex, making the prediction with the model in Eq. (18) less accurate. In fact, the relative errors between the best-fit and measured master curves of VP strain

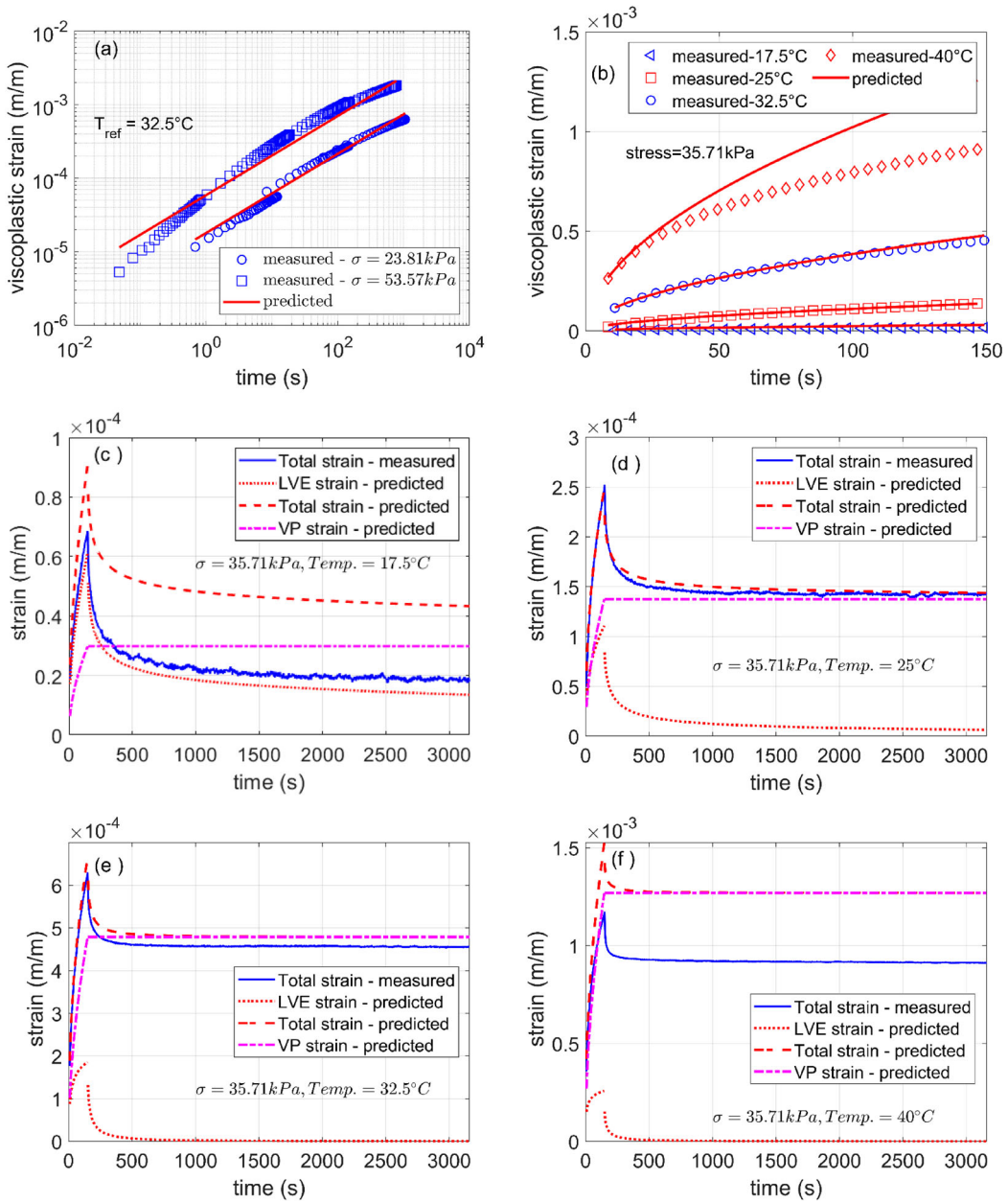


Figure 36. Identifying model parameters and predicting the evolution of VP strains in SS CR tests on AC specimens; (a) determination of model parameters based on the master curve of VP strain at the reference temperature equal to 32.5°C caused by the stress levels equal to 28.81 and 53.57 kPa; (b) prediction of VP strains caused by the stress level equal to 35.71 kPa at various temperatures; (c), (d), (e), and (f) comparison of the predicted total strains with the measured strains at various temperatures.

of PMB and HiMA mixtures are respectively lower than 4.73%, and 9.75% while that of AC mixture exceeds 16.15% despite the high coefficients of determination ($R^2 > 0.974$).

The obtained best-fit parameters were then used to predict the evolutions of VP strains of all mixtures caused by the stress level of 35.71 kPa at various testing temperatures. For that purpose, the *average VP shift factors* determined from tests with the *lowest and highest stress levels* were used to convert the real loading time at a temperature into the equivalent one at the reference temperature,

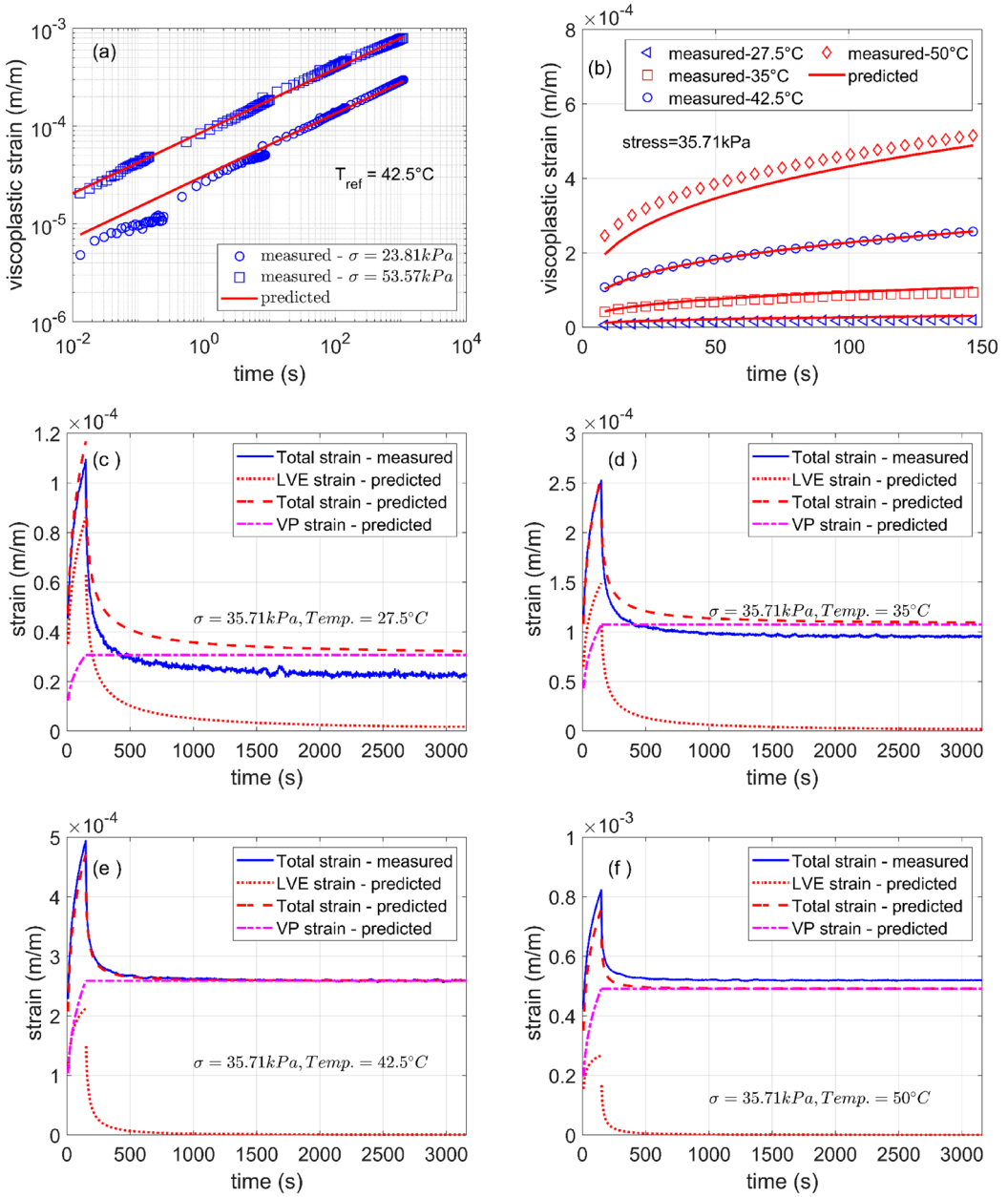


Figure 37. Identifying model parameters and predicting the evolution of VP strains in SSCR tests on PMB specimens; (a) determination of model parameters based on the master curve of VP strain at the reference temperature equal to 42.5°C caused by the stress levels equal to 28.81 and 53.57 kPa; (b) prediction of VP strains caused by the stress level equal to 35.71 kPa at various temperatures; (c), (d), (e), and (f) comparison of the predicted total strains with the measured strains at various temperatures.

called VP reduced time, with which the VP strain at any temperature can be computed:

$$\varepsilon_{VP}(t, T) = A \cdot (t/a_{T,VP}(T))^b \sigma^m. \quad (19)$$

Figures 36–38(b) respectively display the prediction results of VP strains at various testing temperatures of all mixtures. Overall, the relative errors of the prediction were rather small, lower than 12.03% for most cases except that of PMB specimen at 27.5°C and AC specimen at 17.5 and 40°C as reported

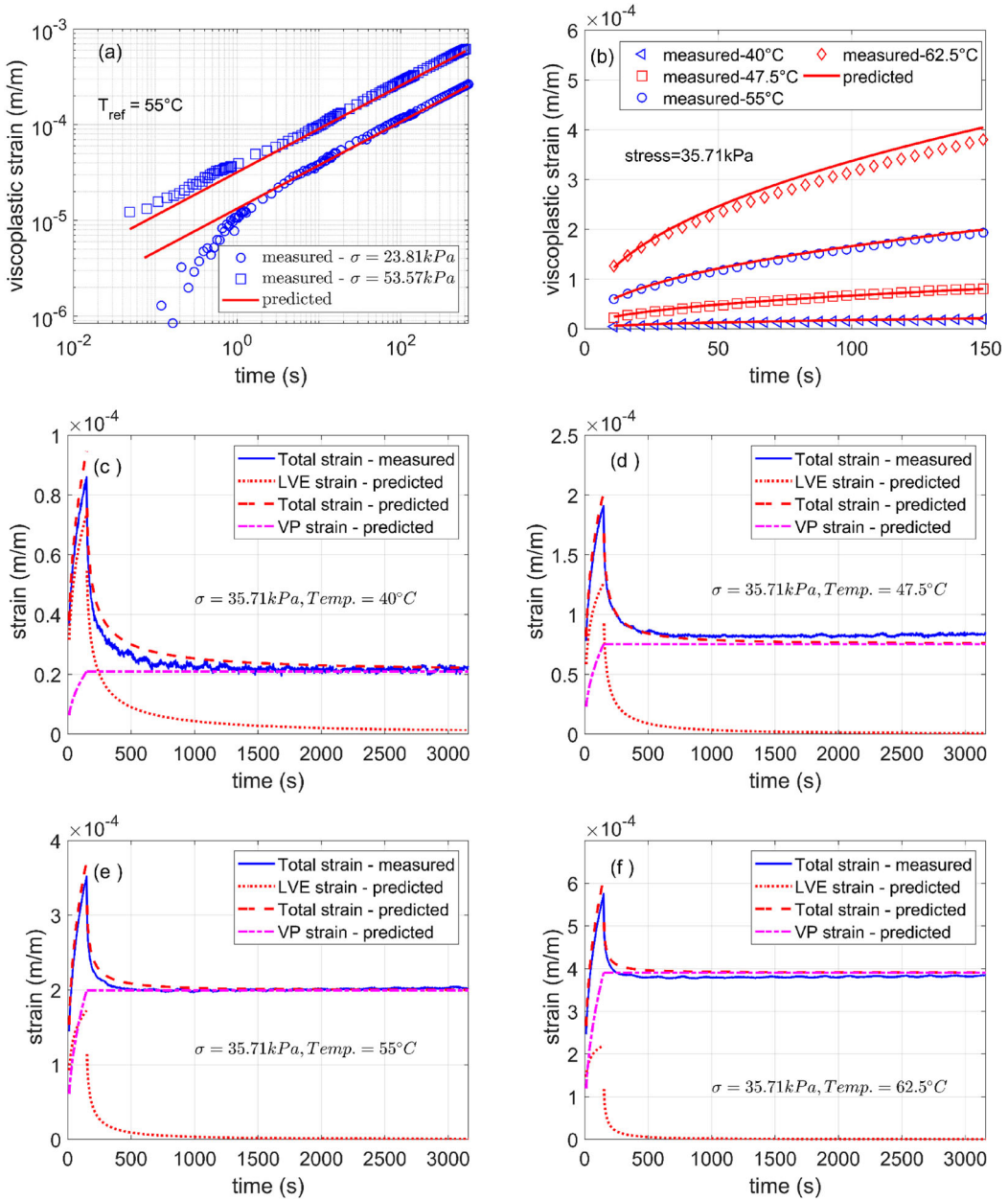


Figure 38. Identifying model parameters and predicting the evolution of VP strains in SSCR tests on HiMA specimens; (a) determination of model parameters based on the master curve of VP strain at the reference temperature equal to 55°C caused by stress levels equal to 28.81 and 53.57 kPa; (b) prediction of VP strains caused by the stress level equal to 35.71 kPa at various temperatures; (c), (d), (e), and (f) comparison of the predicted total strains with the measured strains at various temperatures.

in Table 10. At the lowest testing temperatures, the values of VP strain were really small, at the order of $10^{-5}(\text{m/m})$, which may make the LVDTs more sensitive to sources of perturbation. Meanwhile, the high error of VP prediction of AC mixture at the highest testing temperature can be explained by the simple nature of the VP model used. In fact, the curves of the VP strains of AC mixture in a double logarithmic scale plot are curvilinear while the model is just able to predict a linear relationship.

Table 10. Relative errors of the prediction of VP strains of AC, PMB, and HiMA specimens caused by stress level 35.71 kPa.

AC mixture		PMB mixture		HiMA mixture	
Temp. (°C)	RE (%)	Temp. (°C)	RE (%)	Temp. (°C)	RE (%)
17.5	80.820	27.5	48.724	40.0	12.034
25.0	3.544	35.0	11.188	47.5	6.574
32.5	3.162	42.5	0.848	55.0	3.017
40.0	27.937	50.0	8.499	62.5	2.944

Table 11. Relative errors of the prediction of the total strains of AC, PMB, and HiMA specimens caused by stress level 35.71 kPa.

AC mixture		PMB mixture		HiMA mixture	
Temp. (°C)	RE (%)	Temp. (°C)	RE (%)	Temp. (°C)	RE (%)
17.5	113.405	27.5	38.906	40.0	11.252
25.0	10.085	35.0	28.141	47.5	13.140
32.5	5.076	42.5	1.724	55.0	2.797
40.0	37.703	50.0	5.903	62.5	3.460

Since the VP strains were not directly measured yet back-calculated from the response of the specimens, the accuracy of VP prediction results needs to be verified once more time with the directly measured data, or the total strains. Therefore, the total strains of all mixtures in SSCR tests with the stress level 35.71 kPa at different temperatures were predicted and compared with the measured ones (see Figure 36c-f, 37c-f, and 38c-f). In fact, the VE strains were predicted with *the average CC function* obtained from the recovery data of tests *with the lowest and highest stress levels* and added to the predicted VP strains. As can be observed in Figure 36–38c-f, the predicted and measured total strains agreed well with each other, except that of AC specimen at 17.5°C and 40°C and that of PMB specimen at 27.5°C due to the high errors of the predicted VP strains previously mentioned. In fact, the relative errors between the predicted and measured total strains of all specimens in most cases were rather small, ranging from 1.72 to 28.14% (see Table 11), proving that the constructed master curves of VP strain and their companion VP shift factors indeed characterise the pure PD behaviour of the investigated materials.

Another point to be emphasised is that the predicted VE strains tended to recover all of their values as the resting time increased. Therefore, using only the VE component cannot predict the total strains of the specimens. Furthermore, it can be demonstrated that the predicted VP strain at the end of the loading phase agreed well with the asymptotic values of the measured strains—namely, the relative differences between them were almost lower than 12.52%. This indicates that the permanent strains in the specimens did not evolve during the resting times as assumed and the VP prediction with the best-fit parameters well reflects the evolution of the real permanent deformation in the specimen. Thus, the total strain is indeed comprised of at least two components, a reversible one being the VE strain and an irreversible one mainly constituted by the VP strain as assumed in Section 2.

4. Conclusions

The validity of TTSP in the VP domain within the low-strain regime has been successfully verified via a rigorous testing programme coupled with newly developed theoretical formulations for data analysis. Based on the results obtained, the main features of the present work can be summarised as follows.

- The total strain was a priori assumed to be comprised of VE and VP components, with which the VE and VP behaviour were determined from the results of the SSCR test without performing additional

tests in the frequency domain. Despite that, extra DM tests were also conducted to verify the accuracy of the VE properties determined in the time domain, and the obtained results showed good agreement between the time and frequency domain. In fact, the relative differences between CCs back-calculated and interconverted from DM data were lower than 8.38% for all mixtures while the differences between shift factors were lower than 5.97% for PMB and HiMA mixtures, and equal to 17.94% for AC mixture.

- The proposed shifting method allowed us to objectively construct the master curves of all quantities in the time and frequency domain. The obtained master curves of total and VP strain were relatively smooth and monotonous ensuring an injective relationship between time and the total, or VP strain. In addition, all the shift factors were independent of the stress levels applied and injective with the temperatures. Therefore, TTSP theoretically still held in the VP domain, at least with the examined asphalt mixtures. Furthermore, the validity of TTSP was also experimentally verified within the low-strain regime. In fact, a simple power law VP model was calibrated with the master curves of VP strain obtained from the lowest and the highest stress levels and then used to predict the total strains at a different stress level and temperatures. The prediction achieved a good level of accuracy with the relative errors being almost lower than 28.14% (including inter-sample variations) for the VP strain values up to $1.83 \times 10^{-3} \text{ m/m}$ for AC mixture, $8.01 \times 10^{-4} \text{ m/m}$ for PMB mixture, and $6.25 \times 10^{-4} \text{ m/m}$ for HiMA mixture.
- Based on the prediction results of the total, VE, and VP strain, it was shown that using only one VE component cannot accurately represent the real stress–strain relationship of the materials manifesting in SSCR tests. In addition, the predicted VP strains at the end of the loading phase witnessed good agreement with the asymptotic values of the measured strains as the resting time increased with the relative differences being lower than 12.52% in most cases. Thus, the decomposition assumption made for the total strain was validated.
- Factors causing biases in the obtained results were quantitatively estimated. According to the proposed theoretical formulations, the cut-off time ranges from 6.46 to 15.15 times the loading ramp duration for ensuring an acceptable tolerance of 2% for the VE and VP strain. Regarding the effects of the cumulative permanent contraction strain, it was estimated from replication tests that the total and VP strain suffers from a decrease in value after one cycle of SSCR test, with the reduction ratio being lower than 1.08 for the total strain and 1.25 for the VP strain. As a result, the reliable intervals of shift factors were quantitatively estimated—namely, the largest tolerances of the total and VP shift factors were respectively $\pm 23.17\%$ and $\pm 34.05\%$ of the mean values.
- It was demonstrated that the total, VE, and VP shift factors seem to have similar values at elevated temperatures. The mean RD between the total and VE shift factors was 13.76% for AC mixture, 26.98% for PMB mixture, and 7.04% for HiMA mixture while the mean RD between VP and VE shift factors was 14.95% for AC mixture, 35.71% for PMB mixture, and 11.73% for HiMA mixture. This may explain the suggestion of many authors to extend the validity of VE shift factors into VP domain.
- As the full range of temperatures was considered, obvious independence between VE and total shift factors, and between VE and VP shift factors was observed. In fact, the RDs between the mean values of the total and VE shift factors reached 33.74% for the case of PMB mixture while the values for the VP-VE shift factor difference were even higher with the RD exceeding 82.44% for the same mixture. In considering the discrepancies of all shift factors, the gaps between the total and VE shift factors, and between the VP and VE shift factors are still clearly recognisable with the potential closest distances between the total and VE shift factors, and between the VP and VE shift factor being higher than 11.42% in all cases.
- Within the small strain regime, the stress level almost has no impact on the total and VP shift factors, which is crucial for the applicability of TTSP in the VP domain.

Owing to the results of this work, the VP behaviour of asphalt mixtures can be investigated over a wide range of loading times by conducting a series of SSCR tests with reasonable testing times instead of performing extremely long-time SSCR tests like the LVE behaviour. Once the master curves of VP

strain are constructed, the resistance to PD of asphalt mixtures will be assessed with high sensitivity. For instance, AC mixture was demonstrated to be the most vulnerable to PD, compared to PMB mixture and HiMA mixture. The results obtained in this study are only valid in the small strain regime. For the large strain behaviour, nonlinear VE theory must be implemented in the formulation to replace the Boltzmann superposition principle, which will be carried out in future works.

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Appendix

A1. Estimation of the cut-off time for VE strains (and CCs) obtained from SSCR tests

The objective of this section is to analytically estimate the value of the cut-off time t_c so that the obtained VE strains are considered reliable. Since the time range of interest in this analysis is relatively short, the creep compliance function at all testing temperatures can be approximated by the power law in Equation (A1) to facilitate the analytical investigation:

$$D(t) \approx D_1 t^{\alpha_1}. \quad (A1)$$

The VE strain caused by stress histories 1 and 2 (see Figure 25) at $0 < t \leq t_0$ can be easily computed by:

$$\varepsilon_{VE,1}(t) = D_1 t^{\alpha_1} \cdot \sigma_1. \quad (A2)$$

$$\varepsilon_{VE,2}(t) = \frac{D_1 t^{\alpha_1+1}}{\alpha_1 + 1} \cdot \frac{\sigma_1}{t_0}. \quad (A3)$$

It is clear that $\varepsilon_{VE,2}(t)$ is less than $\varepsilon_{VE,1}(t)$ for $\alpha_1 > 0$ and $t \leq t_0$. As a result, there exists a time, noted t_s , such that the VE strain caused by stress history 1 at t_s is equal to the VE strain caused by stress history 2 at a time t_0 :

$$\frac{t_s}{t_0} = (\alpha_1 + 1)^{-\frac{1}{\alpha_1}}. \quad (A4)$$

Consequently, the ramp loading history can be approximated by a pure creep loading history in which the applied stress is delayed by an amount of time equal to $t'_0 = t_0[1 - (\alpha_1 + 1)^{-1/\alpha_1}]$. With the aid of the equivalent loading history (stress history 3 in Figure 25b), the VE strain caused by the real loading history (history 2) at any time greater than t_0 can be computed:

$$\varepsilon_{VE,2}(t) \approx D_1 (t - t'_0)^{\alpha_1} \cdot \sigma_1. \quad (A5)$$

Let TOL be the acceptable value of the relative tolerance, one can estimate the cut-off time as follows:

$$\frac{t_c}{t_0} = \frac{1 - (\alpha_1 + 1)^{-1/\alpha_1}}{1 - (1 - TOL)^{1/\alpha_1}}. \quad (A6)$$

With the TOL chosen equal to 2%, the estimation of cut-off times for VE strains (and CCs) of all mixtures at all testing temperatures were calculated and are reported in Table A1. According to the data reported, the cut-off time varied with testing temperature and material type, ranging from 6.46 to 11.83 times the loading ramp duration t_0 . However, the value of t_s did not change much and its average value is approximately $2t_0/5$. Thus, the average value of the delayed time t'_0 was nearly $3t_0/5$.

A2. Estimation of the cut-off time for VP strains obtained from SSCR tests

Let us now compute the VP strains caused by the two loading histories as illustrated in Figure 25 using the model in Equation (18):

$$\varepsilon_{VP,1}(t) = A t^b \sigma_1^m, \quad (A7)$$

$$\varepsilon_{VP,2}(t) \approx A \left(t - \frac{3t_0}{5} \right)^b \sigma_1^m. \quad (A8)$$

Table A1. Estimation of cut-off times for VE strains of all mixtures determined for each testing temperature.

Mixture	Temperature (°C)	α_1	t_s/t_0	t_c/t_0
AC	17.5	0.411	0.433	11.830
	25.0	0.378	0.428	10.982
	32.5	0.311	0.419	9.254
	40.0	0.211	0.404	6.535
PMB	27.5	0.304	0.418	9.044
	35.0	0.292	0.416	8.749
	42.5	0.270	0.413	8.156
	50.0	0.223	0.405	6.867
HiMA	40.0	0.328	0.421	9.694
	47.5	0.327	0.421	9.670
	55.0	0.299	0.417	8.917
	62.5	0.208	0.403	6.460

It should be mentioned that the VP model in Equation (18) is only valid for the pure creep loading history. Therefore, it cannot be used to compute the evolution of VP strain in the loading ramp period of stress history 2. Instead, the delayed time t'_0 previously estimated for VE strain was utilised to convert the ramp loading history 2 into an equivalent pure creep loading in the consideration of the VP strain evolution. By applying the same approach, the cut-off times for VP strains can be estimated:

$$\frac{t_{c,VP}}{t_0} = \frac{3}{5} \left[1 - (1 - TOL)^{\frac{1}{b}} \right]^{-1}. \quad (A9)$$

With the same value of the tolerance and the values of b given in Table 9, the estimated cut-off times for VP strains of AC, PMB, and HiMA mixtures respectively equal to 15.15, 9.51, and 13.82 times the loading ramp duration t_0 .

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Characterising The Creep Compliance of A Conventional Asphalt Mixture Using Creep-Recovery Test

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Abstract. Asphalt pavement (or flexible pavement) accounts for a large amount of the road transportation system in Vietnam for its ease of construction and maintenance, and the smoothness it provides to the vehicles while circulating on the pavement. Compared with rigid pavement, asphalt pavement has a shorter service life; thus, more maintenance works are required to ensure the good quality of the pavement. Therefore, understanding the behavior of the material serves as the key point for the appropriate design of the pavement and ensuring a long service life. This paper will present a method for characterizing the linear viscoelastic properties of asphalt mixtures in the time domain, with which deeper insights into the behavior of the material at elevated temperatures can be revealed. The results of this study allow improving the accuracy of the analysis and design of pavement structures and thus the service life of the pavement.

Keywords: Linear viscoelasticity, Creep compliance, Asphalt mixture, Stress and Strain, Time domain.

1 Introduction

Creep compliance (CC) and relaxation modulus (RM) are two linear viscoelastic (LVE) functions in the time domain. Their conjugate functions in the frequency domain are respectively the complex compliance and complex modulus (CM). The four LVE functions are all mathematically interrelated because the LVE behavior of the material is objective with the manner of measurement. However, experiments have shown that the determination of LVE properties in the frequency domain is much easier to perform. The reason is that beyond some level of temperature, asphaltic material exhibits irreversible strain with which the measured strain is not a pure viscoelastic quantity. In fact, Liu and You ([1]) reported that the irreversible deformation of an asphalt mixture in a creep-recovery (CR) test may account for up to 80% of the total deformation when the loading time is larger than 10s at 40°C. Thus, the LVE characterization of asphalt mixture by using relaxation and creep tests in the time domain often leads to large errors at high temperatures. In contrast, the LVE properties of the material determined by the dynamic modulus (DM) test are more reliable because the effect of viscoplastic (VP) strain can be easily eliminated.

In most finite element method (FEM) programs (e.g., ABAQUS, COMSOL) the RM or CC in the form of sums of exponential functions (SEFs) is required to define the LVE behavior of the material. Consequently, the value of the RM or CC must be converted from CM data and decomposed into the SEFs. As a result, small errors in the characterization of CM may enable large deviations in the analysis results in the time domain. Therefore, direct characterization of CC of asphalt mixture may improve the perceiving of the behavior of the material at elevated temperatures. Recently, Nguyen et al. [2] have proposed an algorithm to back-calculate the pure CC based on CR test data, in which the influence of irreversible strain is eliminated and the pure LVE behavior can be extracted.

Therefore, this study aims to assess the applicability of the algorithm proposed in [2] in characterizing the CC of a conventional asphalt (AC) mixture based on CR test data. For that purpose, classical DM tests were also conducted on the same material to determine the corresponding CM. The interconversion method proposed in [3] was used to convert the CM data into the CC functions based on which the agreement between the two quantities, CC determined in both domains (the time and frequency), can be assessed.

2 Material and methods

2.1 Material

This work investigates the LVE properties of a conventional asphalt concrete (AC) that has been widely used in Vietnam. It was produced by mixing a suitable amount of neat bitumen of 60/70 pen (equivalent to performance grade PG 64) with well-graded aggregate that was collected from a quarry in the South of Vietnam. The nominal maximum aggregate size was 12.5mm and the gradation curve was designed based on the Marshall method and the specifications in [4]. The specimen used for all kinds of tests is prismatic with dimensions: $0.07 \times 0.07 \times 0.15 \text{ m}^3$, which was sawed from a compacted plate of the AC mixture with dimensions: $0.32 \times 0.26 \times 0.018 \text{ m}^3$.

2.2 Characterizing the creep compliance by using creep-recovery tests

The applied load and the response deformation of CR tests are illustrated in Figure 1. The period without applied load is the unloading one wherein only the recovery of viscoelastic (VE) strain occurs. In this study, to directly determine the CCs of the AC mixture, the algorithm proposed by Nguyen et al. [2] was used for its efficiency in processing CR test data even at intermediate and elevated temperatures.

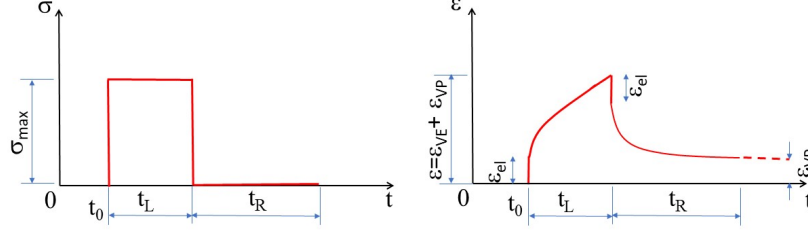


Fig. 1. Illustration of the applied load and the response deformation in a CR test.

2.3 Time-frequency interconversion of LVE properties

To verify the accuracy of the CCs determined from CR tests, the DM tests were performed, and the obtained CM data were then converted into CCs in the time domain by using the method proposed by Phan et al. ([3]).

3 Results

3.1 Results of CR tests

The data of the unloading period of CR tests and results of back-calculated CCs at various temperatures (-5, 2.5, 10, 20, 30, and 40°C) are presented in this subsection. To ensure that the material is still working in the LVE domain, the applied stress levels were carefully chosen so that the response strains did not exceed $150\mu\epsilon$ in all tests (see Table 1). The loading and resting times of each test are 150s and 750s, respectively.

The best-fit solutions of all tests are shown in Figure 2a and the CCs of the material were computed and are presented in Figure 2b. By adopting the time-temperature superposition principle (TTSP), the CCs at all temperatures were shifted along the time axis to construct their master curve (MSC) (see Figure 3a). The reference temperature was chosen $T_{ref} = 20^\circ\text{C}$, and the corresponding shift factors a_T are shown in Figure 3b.

Table 1. Applied stresses on the sample in CR tests.

Temperature (°C)	-5	2.5	10	20	30	40
Stress (kPa)	0.612	0.510	0.408	0.065	0.018	0.012

Table 2. Parameter of the HS model in this study.

E_0 (MPa)	E_∞ (MPa)	k	h	δ	τ
80	34,500	0.163	0.480	1.504	0.029

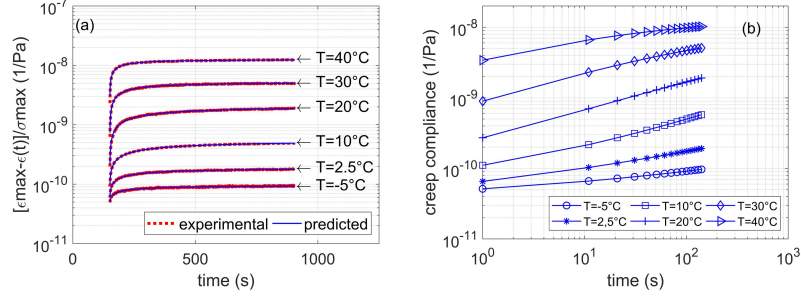


Fig. 2. (a) Best-fit solutions and (b) back-calculated CCs of the study mixture at all testing temperatures.

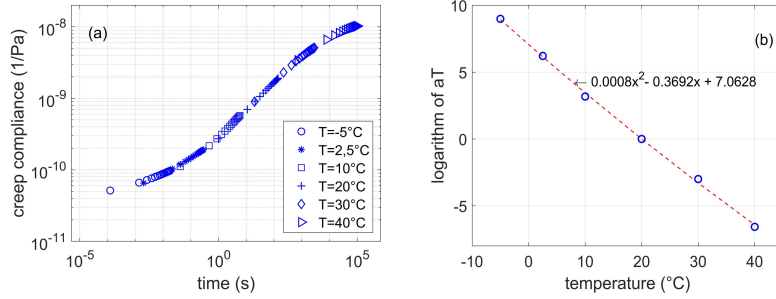


Fig. 3. Master curve of back-calculated CCs (a) and corresponding shift factors (b).

3.2 Results of dynamic modulus test

Temperatures of DM tests were chosen the same as those in CR tests. At each temperature, testing frequencies were swept from the lowest to the highest as follows: 0.033, 0.1, 0.33, 1, and 3.33Hz. The applied stresses were carefully chosen so that the strain magnitudes were lower than $75\mu\epsilon$ to ensure that the behavior of the samples still stayed within the LVE regime. The obtained data were processed based on the specifications in [5]. The MSC of DM was constructed using the horizontal shifting technique and is shown in Figure 4a. By using the same shift factors, the MSC of phase angles was also constructed as presented in Figure 4b.

The LVE behavior of the AC mixture can be predicted by either a mathematical or a rheological model. In this study, the Huet-Sayegh (HS) model, a rheological one, is chosen because the model involves fewer parameters yet well predicts the LVE behavior of asphaltic materials. The best fitting of the measured dynamic moduli and phase angles was plotted in Figures 4a-b with the HS parameters displayed in Table 2. As can be observed, the HS model can predict very well the LVE properties of the studied mixture. In fact, the relative error between the measured and the predicted values is just 4.095% for dynamic moduli and 6.329% for phase angles. In addition,

Cole–Cole and Black diagrams in Figure 4c-d also confirm the good level of fitting by the HS model.

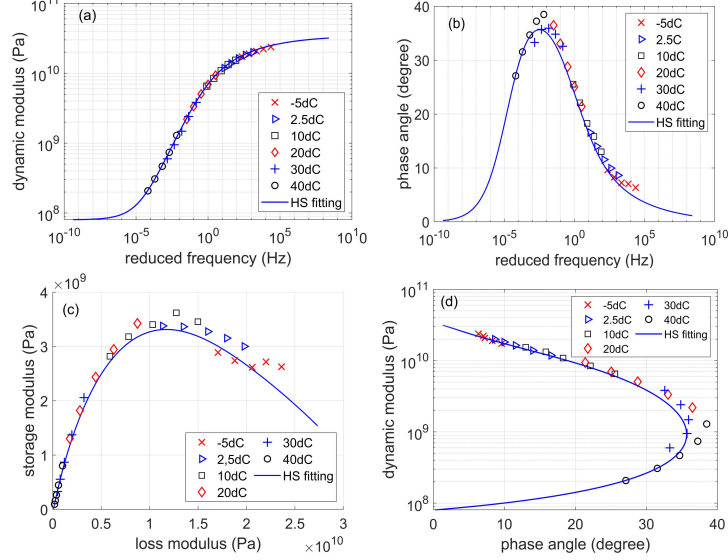


Fig. 4. Results of LVE characterization in the frequency domain. (a) Dynamic moduli (b) phase angles, (c) Cole – Cole diagram, and (d) Black diagram.

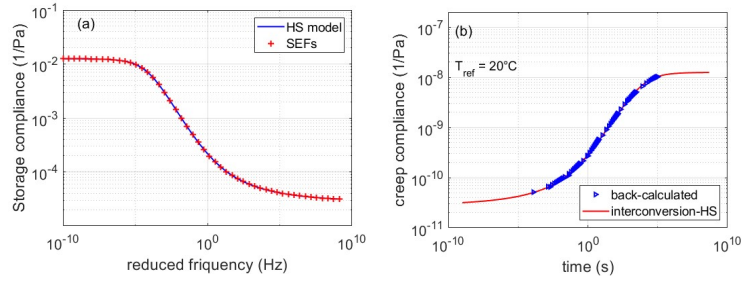


Fig. 5. Interconversion of dynamic modulus data into creep compliance. (a) Result of approximation of storage compliance by SEFs, (b) MSC of back-calculated CCs and interconverted CC.

3.3 Results of time-frequency interconversion

First, the storage compliance was decomposed into SEFs with the aid of the modified Dombi method ([4]) as shown in Figure 5a. With the order of the polynomial chosen equal to 2000, the relative errors of the decomposition were extremely small, lower than 1.537×10^{-6} at all computed points. Then, the CC function was converted from

the storage compliance and is plotted with the CCs determined from CR tests in Figure 5b.

The MSC of the converted and back-calculated CCs are in full accord with the relative difference between them being lower than 1.6%. It means that the MSC of back-calculated CCs is reliable.

4 Conclusions

The LVE properties of a conventional asphalt mixture have been successfully characterized in the time domain even at elevated testing temperatures. It was demonstrated that the CCs determined in the time domain agree well with that interconverted from CM data. Consequently, the behavior of an LVE material can be fully characterized in the time domain without passing through the frequency domain. This allows the prevention of intermediate errors encountered in the prediction of LVE behavior in the frequency domain as well as in the interconversion phase. Thus, the results of this work could be applied to improve the accuracy of the LVE characterization in the time domain as well as the analysis and design of pavement structures which are normally run in the time domain.

Acknowledgment

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- Tập 2. Động lực học và Điều khiển; Cơ học Thủy khí; Cơ học Tính toán; Cơ học Máy

Ban Tổ chức cảm ơn sự tham gia nhiệt tình của các thành viên Ban tổ chức, Ban Thư ký và Ban khoa học hội nghị. Ban Tổ chức xin cảm ơn các đơn vị: Trường Đại học Công nghệ (ĐHQGHN), Viện Nghiên cứu Cơ khí, Trường Đại học Xây dựng Hà Nội, Trường Đại học Sư phạm kỹ thuật Vĩnh Long đã nhiệt tình ủng hộ, tài trợ cho Hội nghị.

Mong nhận được ý kiến đóng góp của bạn đọc về nội dung và hình thức Tuyển tập.

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So sánh độ chính xác của các phương pháp xấp xỉ bằng tổng của hàm số mũ sử dụng các hệ số có giá trị thực

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Tóm tắt. Trong bài viết này, nhóm tác giả đánh giá các phương pháp thông dụng dùng xấp xỉ một hàm số thành tổng của các hàm số mũ như phương pháp hồi quy phi tuyến, phương pháp collocation và một phương pháp mới được giới thiệu gần đây là phương pháp Dombi cải tiến thông qua các chỉ số về độ chính xác và sự hội tụ và chi phí tính toán. Kết quả nghiên cứu cho thấy phương pháp Dombi cải tiến có độ chính xác cao nhất, sự hội tụ nhanh nhất nhưng lại tốn nhiều thời gian tính toán nhất. Mặc dù vậy, thời gian tính toán là chấp nhận được để có thể đạt được mức sai số tương đối nhỏ hơn một phần triệu ở tất cả các điểm khảo sát mà không cần phải chọn trước các hệ số, điều mà các phương pháp còn lại không thể đạt được.

Từ khóa: Phương pháp xấp xỉ, tổng của hàm số mũ, phương pháp Dombi cải tiến, phương pháp collocation, hồi quy phi tuyến.

1. Mở đầu

Hàm số có dạng là tổng của các hàm số mũ như được biểu diễn theo phương trình (1)

$$f(x) = a_0 + \sum_{j=1}^m a_j e^{b_j x} \quad (1)$$

được ứng dụng rộng rãi trong nhiều lĩnh vực kỹ thuật. Vì dạng hàm này có thể dễ dàng tích phân hoặc vi phân nên các nhà nghiên cứu có thể tìm ra được các nghiệm cho nhiều vấn đề kỹ thuật. Có thể kể đến một số bài toán như theo dõi số lượng vi sinh vật trong môi trường nuôi cấy trong lĩnh vực sinh vật hay các thuật toán phức tạp trong lĩnh vực công nghệ máy tính đòi hỏi tài nguyên như thời gian tính toán, bộ nhớ tăng theo cấp số nhân.

Dạng hàm này cũng được sử dụng một cách hiệu quả trong lĩnh vực phân tích và thiết kế kết cấu bê tông nhựa. Ứng xử của vật liệu bê tông nhựa được xem như vật liệu đàn hồi nhớt vì vậy để dự đoán các ứng xử của vật liệu, nhiều loại mô hình dự báo đã được xây dựng bao gồm các mô hình lưu biến và mô hình toán học. Trong các mô hình ứng xử này, các mô hình lưu biến như Huet-Sayegh (HS), 2S2P1D được sử dụng khá rộng rãi do khả năng khớp rất tốt với các dữ liệu thí nghiệm với tương đối ít các tham số mô hình (Di Benedetto và cộng sự, 2004; Pronk, 2005). Tuy nhiên, các mô hình này lại khó có thể tích hợp vào các chương trình phần tử hữu hạn tính toán trong miền thời gian do chúng không có sẵn hàm đạo ở dạng tường minh (Nguyễn và cộng sự, 2020). Theo Nguyễn và cộng sự (2020), nếu hàm đạo của vật liệu đàn hồi nhớt tuyến tính có dạng:

$$E(t) = E_0 + \sum_{j=1}^{N_j} a_j e^{b_j(t/(a_r(T)))} \quad (2)$$

thì mối quan hệ giữa ứng suất và biến dạng của vật liệu có thể được xác định bởi một công thức qui nạp. Nghĩa là mối quan hệ giữa ứng suất và biến dạng của vật liệu chỉ liên quan đến các đại lượng tại thời điểm tính toán và thời điểm trước đó. Kết quả này giúp giảm một cách đáng kể tài

nguyên và thời gian tính toán khi phân tích và thiết kế kết cấu bê tông nhựa.

Từ ý nghĩa thiết thực của hàm có dạng tổng các hàm số mũ, một số phương pháp xấp xỉ một hàm bất kỳ bằng tổng các hàm số mũ đã được đề xuất và áp dụng. Schapery (1962) đã dùng phương pháp collocation để khớp với hàm đạo của các vật liệu polyme. Phương pháp này được Park và Kim (2001) cải tiến bằng cách làm trơn trước dữ liệu cần khớp và bổ sung các chuỗi hàm đa thức để làm giảm sai số của phương pháp. Bên cạnh phương pháp collocation, phương pháp hồi quy phi tuyến cũng được Luo, Lv và Liu (2018) sử dụng để xấp xỉ hàm độ mềm từ biến của vật liệu bê tông nhựa. Đến những năm gần đây, Dombi (2006) đã giới thiệu một thuật toán mới để xấp xỉ một hàm bằng tổng các hàm số mũ và thuật toán này đã được Nguyen và cộng sự (2020) cải tiến, được gọi là phương pháp Dombi cải tiến, nhằm xấp xỉ hàm đạo của vật liệu nhựa đường và bê tông nhựa. Mức độ sai số của quá trình xấp xỉ có ảnh hưởng rất lớn đến các bước tính toán phía sau. Vì vậy, mục tiêu của bài báo này là so sánh độ chính xác, sự hội tụ và thời gian tính toán nhằm đánh giá mức độ hiệu quả của ba phương pháp: phương pháp collocation, phương pháp hồi quy phi tuyến và phương pháp Dombi cải tiến. Nghiên cứu này là cơ sở giúp cho các nhà khoa học lựa chọn phương pháp xấp xỉ một hàm số bằng tổng các hàm số mũ phù hợp với mục tiêu của nghiên cứu.

2. Cơ sở lý thuyết

2.1. Phương pháp hồi quy phi tuyến

Phương pháp hồi quy phi tuyến có thể được dùng để xấp xỉ một hàm số bằng một hàm số được chọn trước. Cũng giống như phương pháp hồi quy tuyến tính, quy trình xấp xỉ của phương pháp hồi quy phi tuyến cũng là xác định các tham số của hàm được chọn sao cho tổng bình phương khoảng cách giữa hai hàm là bé nhất. Nếu ký hiệu y_i^1 là giá trị của hàm được xấp xỉ và y_i^2 là giá trị của hàm được chọn tại điểm x_i thì mục tiêu của phương pháp hồi qui phi tuyến là cực tiểu hóa tổng bình phương số dư:

$$SS = \sum_i (y_i^1 - y_i^2)^2. \quad (3)$$

Hàm được chọn trước trong phương pháp hồi quy phi tuyến ở dạng tổng quát có thể biểu diễn như sau:

$$y_i^2 = f(x_i, \Phi) + \varepsilon_i, \quad i = 1, \dots, n \quad (4)$$

trong đó, y_i^2 là các giá trị được xác định từ hàm được xấp xỉ tại các giá trị x_i , f là hàm chọn trước với biến x_i và các tham số Φ , và ε_i là sai số ngẫu nhiên tại x_i .

Ở dạng tổng của các hàm số mũ, hàm chọn trước được biểu diễn như sau:

$$f(x_i, \Phi) = \phi_1 + \sum_{j=1}^m \phi_{j+1} e^{-\phi_{j+m+1} x_i}, \quad (5)$$

trong đó m là tổng số hàm mũ sử dụng. Để đơn giản hóa việc ghi chú, hàm số trong phương trình (5) được viết gọn lại thành:

$$\bar{f}(x_i) = c + \sum_{j=1}^m a_j e^{b_j x}. \quad (6)$$

Trong lĩnh vực kỹ thuật, phương trình (6) thường được gọi là chuỗi Prony.

2.2. Phương pháp collocation

Khi sử dụng phương pháp collocation để xấp xỉ một hàm số bằng tổng của các hàm số mũ thì hàm xấp xỉ được biểu diễn thành chuỗi Prony với hệ số tự do bằng không như sau:

$$\bar{f}(x_i) = \sum_{j=1}^m a_j e^{b_j x} . \quad (7)$$

trong đó: số lượng và giá trị của b_j được chọn trước. Sau đó, các giá trị a_j được xác định thông qua việc giải hệ các phương trình tuyến tính, công việc này đòi hỏi cần phải có ít nhất $2m$ giá trị của hàm số trong đoạn cần xấp xỉ $x = [x_1, x_n]$. Trong điều kiện thực tế của các bài toán kỹ thuật thì số lượng giá trị của hàm số thường lớn hơn nhiều so với các hệ số cần tìm của hàm xấp xỉ, khi đó phương pháp bình phương cực tiểu được sử dụng để tìm nghiệm của hệ phương trình tuyến tính.

2.3. Phương pháp Dombi và phương pháp Dombi cải tiến

2.3.1. Phương pháp Dombi

Phương pháp này được Dombi giới thiệu gần đây (Dombi, 2006). Nội dung của phương pháp được tóm lược như sau. Giả sử khoảng cần xấp xỉ, $x = [0, x_n]$, được chia thành n khoảng bằng nhau và bằng Δx . Giá trị xấp xỉ của hàm $f(x)$ tại điểm $x_i = i\Delta x$ bằng:

$$\bar{f}(x_i) = \sum_{j=1}^m a_j e^{b_j(i\Delta x)} = \sum_{j=1}^m a_j \left(e^{b_j \Delta x} \right)^i . \quad (8)$$

Đặt $q_j = e^{b_j \Delta x}$ thì hệ phương trình (8) có thể viết lại như sau:

$$\bar{f}(x_i) = \sum_{j=1}^m a_j q_j^i, (i = 0, \dots, n) , \quad (9)$$

trong đó a_j và q_j là các tham số cần xác định.

Giả sử q_j là nghiệm của một đa thức bậc m (với $2m \leq n$) có dạng như sau:

$$q^m + s_1 q^{m-1} + s_2 q^{m-2} + \dots + s_m = 0 , \quad (10)$$

trong đó: $s_1, s_2, \dots, s_m$ là các hệ số của đa thức bậc m . Nếu nhân hai vế của các hàng từ 1 cho đến m với các giá trị $s_m, s_{m-1}, \dots, s_1$ của hệ phương trình (9) và chú ý đến hệ quả của phương trình (10) sẽ thu được phương trình sau: $\bar{f}(x_0)s_m + \bar{f}(x_2)s_{m-1} + \dots + \bar{f}(x_{m-1})s_1 + \bar{f}(x_m) = 0$. Thực hiện tương tự cho các bộ phương trình còn lại, ta thu được hệ phương trình:

$$\begin{cases} \bar{f}(x_0)s_m + \bar{f}(x_2)s_{m-1} + \dots + \bar{f}(x_{m-1})s_1 + \bar{f}(x_m) = 0 \\ \dots \\ \bar{f}(x_m)s_m + \bar{f}(x_{m+1})s_{m-1} + \dots + \bar{f}(x_{2m-1})s_1 + \bar{f}(x_{2m}) = 0 \\ \dots \\ \bar{f}(x_{n-m})s_m + \bar{f}(x_{n-m+1})s_{m-1} + \dots + \bar{f}(x_{n-1})s_1 + \bar{f}(x_n) = 0 \end{cases} . \quad (11)$$

Thực tế, số lượng phương trình trong hệ phương trình (11) sẽ nhiều hơn số ẩn cần tìm nên nghiệm

của hệ có thể thu được bằng phương pháp bình phương cực tiểu. Nghĩa là các hệ số của phương trình (10) được xác định. Bằng cách sử dụng các thuật toán tìm nghiệm đa thức ví dụ như giải thuật tìm kiếm giá trị riêng của ma trận đồng hành được lập trình trong Matlab, nghiệm của phương trình (10) sẽ được xác định. Từ đó, ta có thể tìm được các hệ số mũ dùng trong tổng các hàm số mũ:

$$b_j = \frac{\ln q_j}{\Delta x}. \quad (12)$$

Cần lưu ý rằng các hệ số bao gồm cả phần thực và phần ảo và có thể lặp lại nên ta cần loại bỏ các phần ảo và giá trị lặp lại. Sau khi đã xác định được các hệ số mũ, các hệ số a_j sẽ được xác định bằng phương pháp bình phương cực tiểu như đã trình bày bên trên.

3.2.2. Phương pháp Dombi cải tiến

Do phải chia đoạn khảo sát thành n khoảng bằng nhau nên phương pháp Dombi sẽ tốn nhiều thời gian tính toán hoặc giảm độ chính xác khi phải xấp xỉ một hàm có biến khảo sát có khoảng giá trị rất rộng. Nguyen và cộng sự (2020) đã nhận ra vấn đề này và đã đề xuất cải tiến phương pháp Dombi như sau:

(i) Chia biến cần khảo sát thành các khoảng nhỏ và xác định các hệ số mũ cho mỗi khoảng nhỏ đó bằng phương pháp Dombi,

(ii) Loại bỏ các giá trị hệ số trùng lặp và ghép nối các hệ số mũ còn lại thành một tập hợp các hệ số mũ duy nhất.

(iii) Xác định các hệ số a_j của các hàm số mũ bằng kỹ thuật bình phương cực tiểu trên toàn miền cần khảo sát.

3. Kết quả tính toán

3.1. Kết quả xấp xỉ với một hàm có dạng tổng của các hàm số mũ

Thực hiện xấp xỉ hàm số dưới đây

$$f(x) = e^{1.01x} + 0.005e^{0.5x} \quad (13)$$

với ba phương pháp trình bày bên trên trong khoảng $x = [-1, 9]$ được chia đều thành 100 đoạn với 101 giá trị của x và $f(x)$ tương ứng. Các giá trị bắt đầu của quá trình xấp xỉ bằng phương pháp hồi quy phi tuyến được lấy ngẫu nhiên bằng 1. Đối với phương pháp collocation, các giá trị b_j được lấy trong khoảng $[-1, 3]$ nhằm đảm bảo khả năng xấp xỉ của phương pháp, đồng thời tránh trường hợp các giá trị b_j được chọn cho phương pháp collocation ngẫu nhiên trùng khớp với nghiệm chính xác của hàm đầu vào. Các hàm xấp xỉ được khảo sát đến tổng của 6 hàm số mũ. Kết quả xấp xỉ tốt nhất và sự hội tụ của các phương pháp được thể hiện ở Hình 1 và các hệ số tương ứng được trình bày trong Bảng 1. Sai số tương đối của quá trình xấp xỉ trong báo cáo này được định nghĩa như sau:

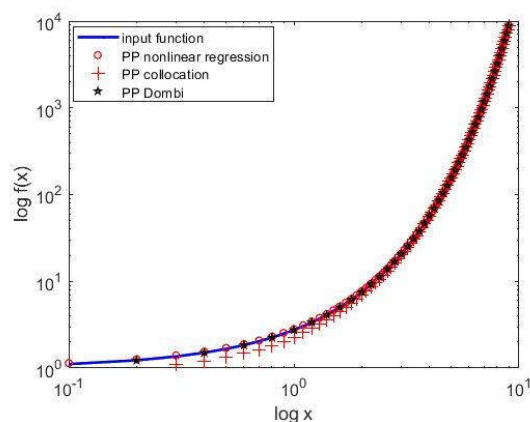
$$err = \max \left(\frac{|\overline{f_i} - f_i|}{f_i} \right), \text{ với } i = 1..n \quad (14)$$

trong đó: $\overline{f_i}, f_i$ lần lượt là giá trị của hàm xấp xỉ và hàm được xấp xỉ tại điểm x_i .

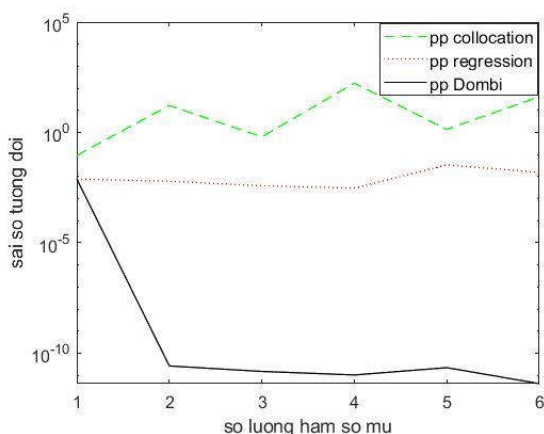
So sánh độ chính xác của các phương pháp xấp xỉ bằng tổng của hàm số mũ sử dụng các hệ số có giá trị thực

Bảng 1. Các hệ số a_j và b_j ứng với trường hợp hàm khảo sát là tổng của 6 hàm số mũ

STT	Hồi quy phi tuyến		Collocation		Dombi	
	a_j	b_j	a_j	b_j	a_j	b_j
1	1.6403818E-01	9.9268133E-01	1.8874444E+01	-1.0000000E+00	1.0000000E+00	1.0100000E+00
2	7.0583207E-02	9.5400359E-01	-3.2241241E+01	-2.0000000E-01	5.0000000E-03	5.0000000E-01
3	2.1669430E-01	1.0008525E+00	6.6479911E+00	6.0000000E-01	0.0000000E+00	-2.9330714E+00
4	2.6121447E-01	9.9311680E-01	4.6814254E-02	1.4000000E+00	-1.2798107E-12	-2.9330714E+00
5	1.3772513E-01	1.0377330E+00	-2.7147526E-05	2.2000000E+00	6.3660584E-13	-3.6171158E+00
6	1.6862275E-01	1.0377267E+00	8.1294074E-09	3.0000000E+00	0.0000000E+00	-3.6171158E+00



(a) Kết quả xấp xỉ



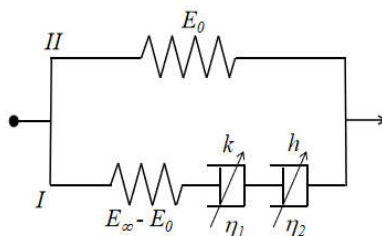
(b) Khảo sát sự hội tụ

Hình 1. Kết quả xấp xỉ và sự hội tụ của ba phương pháp khảo sát đối với dạng hàm là tổng của các hàm số mũ

Kết quả cho thấy khi xấp xỉ một hàm số ở dạng tổng của các hàm số mũ bằng một hàm cũng có dạng tổng của các hàm số mũ, trong ba phương pháp nêu trên, phương pháp Dombi là có khả năng xấp xỉ tốt nhất. Về độ hội tụ, phương pháp collocation là phương pháp kém nhất khi gần như không thể xấp xỉ tốt hàm đầu vào. Điều này là hoàn toàn hợp lý vì khi sử dụng phương pháp này, các hệ số mũ phải được chọn trước. Do đó, trong trường hợp số lượng các hàm số mũ bị giới hạn, khả năng xấp xỉ của phương pháp sẽ bị hạn chế.

3.2. Kết quả xấp xỉ với hàm môđun dẫn của mô hình HS

Mô hình lưu biến Huet - Sayegh (Huet, 1963; Sayegh, 1965) được sử dụng rộng rãi trong dự đoán ứng xử của vật liệu bê tông nhựa. Mô hình bao gồm một lò xo mắc song song với một cụm gồm một lò xo và hai cản nhớt phi tuyến. Mô hình được minh họa ở Hình 2.



Hình 2. Minh họa mô hình Huet - Sayegh (HS)

Trong mô hình HS, các đại lượng đặc trưng của vật liệu được xác định trong miền tần số. Khi áp dụng một biến dạng có dạng $\varepsilon = \varepsilon_0 e^{i\omega t}$ thì ứng suất đáp ứng sẽ có dạng $\sigma = \sigma_0 e^{i\omega t + \varphi}$, với $\omega(\text{rad/s})$ là tần số góc và φ là góc lệch pha. Mô đun phức của vật liệu thu được từ mô hình HS như sau (Olard và Di Benedetto, 2003; Pronk, 2005):

$$E^*(\omega) = E_0 + \frac{E_\infty - E_0}{1 + \delta_1 (i\omega\tau)^{-k} + \delta_2 (i\omega\tau)^{-h}}, \quad (15)$$

trong đó: δ, h, k là các thông số của mô hình và E_0, E_∞ là giá trị của $E^*(\omega)$ khi $\omega\tau \rightarrow 0$ và $\omega\tau \rightarrow \infty$.

Trong ví dụ này, ba phương pháp được giới thiệu ở phần trên được dùng để xấp xỉ hàm mô đun dãn của vật liệu bê tông nhựa đã được mô hình hóa bằng mô hình HS với các thông số của mô hình được lấy từ nghiên cứu của Pronk (2005) và được tóm tắt ở Bảng 1.

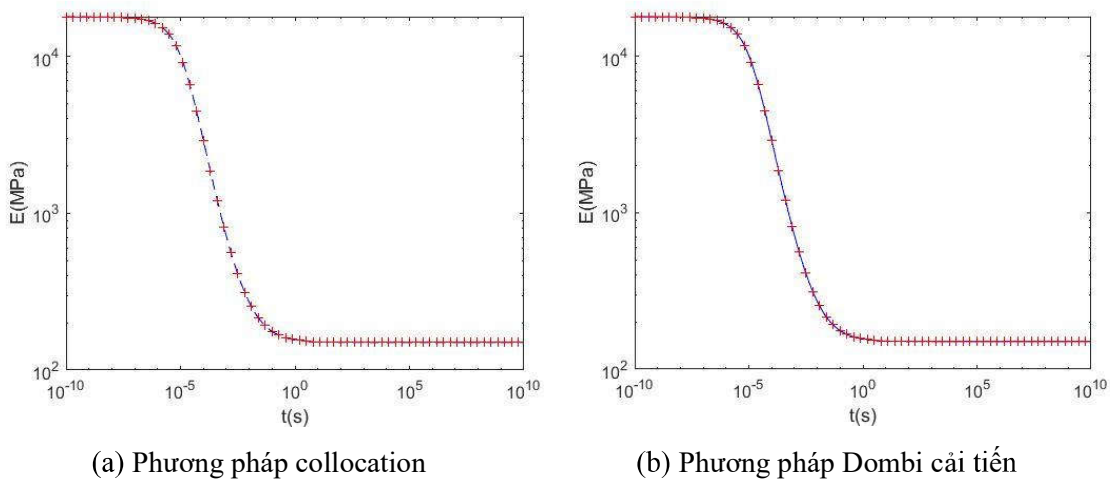
Bảng 2. Thông số của mô hình HS lấy từ nghiên cứu (Pronk, 2005)

$E_0 \text{ (MPa)}$	$E_\infty \text{ (MPa)}$	k	h	δ	$\tau \text{ (s)}$	T_{ref}
150	18.000	0,224	0,656	0,083	$1,334 \times 10^{-5}$	15°C

Do hàm dãn là dạng hàm giảm nên trong ví dụ này, phương pháp collocation sẽ được khảo sát với các giá trị b_j được lấy trong khoảng $[-10^{-5}, \dots, -1, \dots, -10^{+5}]$, $[-10^{-10}, \dots, -1, \dots, -10^{+10}]$ và $[-10^{-20}, \dots, -1, \dots, -10^{+20}]$ nhằm đánh giá ảnh hưởng của khoảng khảo sát đến độ chính xác của phương pháp. Đối với phương pháp hồi quy phi tuyến, việc xấp xỉ sẽ được thực hiện với khả năng tối đa của phần mềm Matlab.

Các hàm mô đun dãn là các hàm đặc trưng cho tính chất của các vật liệu đàn hồi nhớt được xác định theo biến thời gian với khoảng giá trị rất rộng, từ các giá trị rất bé, ví dụ như 10^{-8} s, cho đến giá trị rất lớn như 10^8 s hoặc thậm chí 10^{12} s. Do biến khảo sát của hàm mô đun dãn rất rộng nên phương pháp Dombi cải tiến được sử dụng để so sánh với hai phương pháp còn lại.

Kết quả xấp xỉ tốt nhất của các phương pháp được thể hiện ở Hình 3.



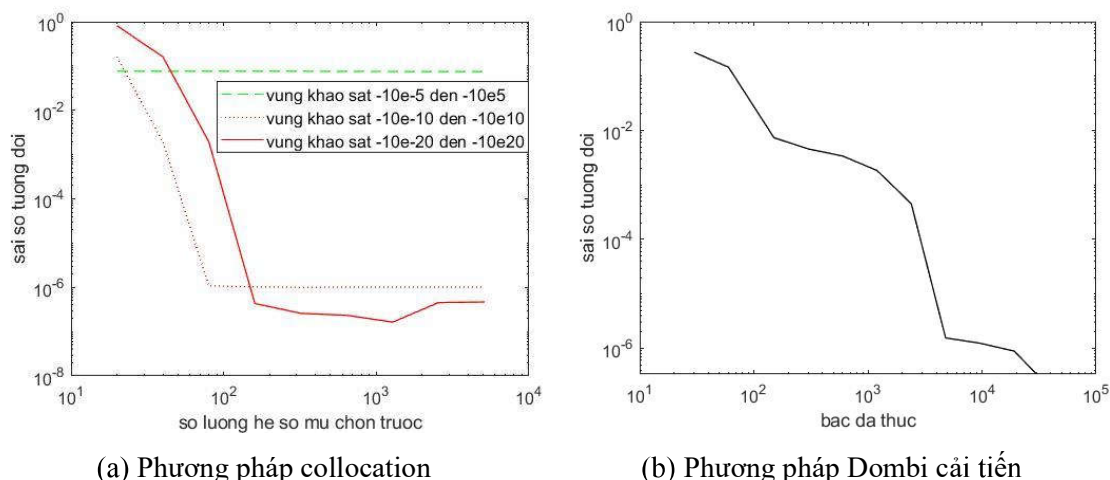
Hình 3. Kết quả xấp xỉ của phương pháp collocation và phương pháp Dombi cải tiến

Trong ví dụ này, phương pháp hồi quy phi tuyến không thể xấp xỉ hàm dãn của vật liệu bê tông nhựa với độ chính xác có thể chấp nhận được. Do đồng thời phải xác định cả hệ số mũ và hệ

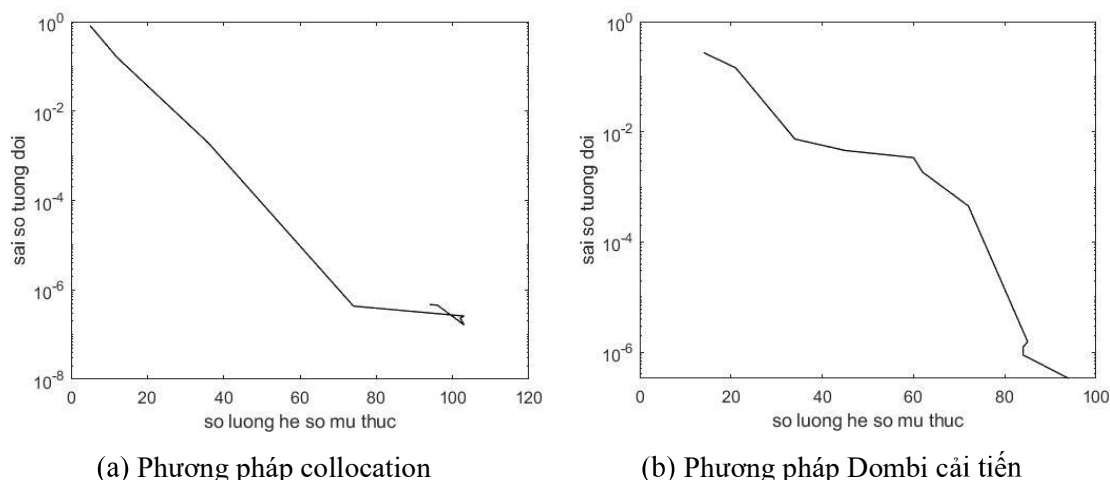
So sánh độ chính xác của các phương pháp xấp xỉ bằng tổng của hàm số mũ sử dụng các hệ số có giá trị thực

số của hàm số mũ nên thường không thể xấp xỉ được các hàm môđun dao với khoảng khảo sát quá rộng. Vì vậy, kết quả thu được với phương pháp này sẽ không được trình bày trong báo cáo này.

Kết quả sự hội tụ của hai phương pháp collocation và Dombi được thể hiện ở Hình 4 và Hình 5.



Hình 4. Sự hội tụ của phương pháp collocation và phương pháp Dombi cải tiến theo bậc đa thức chọn trước/số lượng hệ số mũ chọn trước

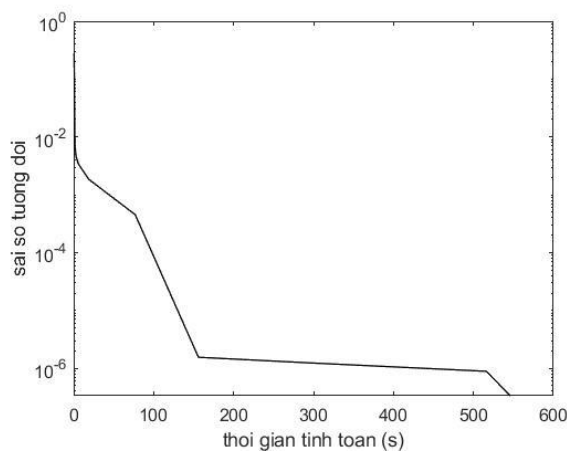


Hình 5. Sự hội tụ của phương pháp collocation và phương pháp Dombi cải tiến theo số lượng hệ số mũ thực

Kết quả cho thấy phương pháp collocation và Dombi cải tiến có sai số tương đối là khá giống nhau ($4,68 \times 10^{-7}$ và $3,35 \times 10^{-7}$) khi vùng khảo sát của phương pháp collocation được lấy từ $[-10^{-20}, \dots, -1, \dots, -10^{+20}]$. Tuy nhiên, nếu chọn vùng khảo sát là $[-10^{-5}, \dots, -1, \dots, -10^{+5}]$ thì sai số của phương pháp collocation là rất lớn so với phương pháp Dombi ($7,52 \times 10^{-2}$ và $3,35 \times 10^{-7}$). Bên cạnh đó, từ Hình 4 và Hình 5 có thể thấy rằng, độ chính xác của phương pháp collocation gần như đã đạt đến giới hạn vì dù số lượng hệ số mũ chọn trước tăng lên thì độ chính xác của phương pháp gần như không được cải thiện. Điều này có thể được giải thích như sau. Khi số lượng hệ số mũ giả định được tăng lên đến một mức độ nào đó thì số lượng các hệ số mũ không phù hợp sẽ tăng

lên. Khi đó các hệ số mũ không phù hợp này sẽ làm nhiễu loạn quá trình tối ưu các hệ số để cực tiểu hóa sai số của hàm xấp xỉ. Kết quả là số hệ số khác không tìm được bị giảm xuống và sai số tương đối giữa hàm xấp xỉ và hàm đầu vào không giảm thêm. Trong khi đó các hệ số mũ tìm được bằng phương pháp Dombi cải tiến đều là những giá trị phù hợp nên việc gia tăng số lượng hệ số mũ không gây ảnh hưởng đến quá trình tối ưu hóa các hệ số nên độ chính xác của phương pháp này tiếp tục được cải thiện.

Về mặt thời gian tính toán, phương pháp collocation giả thiết trước các giá trị của các hệ số mũ nên không tốn nhiều thời gian tính toán, cụ thể thời gian tính toán trong các ví dụ được nghiên cứu trong bài báo này dao động từ 0,1 s – 0,7 s khi thực hiện với bộ xử Intel core i7. Đối với phương pháp Dombi và Dombi cải tiến, thuật toán bao gồm 2 bước, đó là tìm hệ số mũ phù hợp và tối ưu hóa các hệ số nên thời gian tính toán dài hơn, dao động trong khoảng từ 2 đến 9 phút với cùng bộ xử lý. Tuy nhiên, phương pháp Dombi và Dombi cải tiến luôn đảm bảo kết quả xấp xỉ có mức độ sai số theo mong muốn trong khi phương pháp collocation chỉ có thể đạt được điều này nếu các hệ số mũ được lựa chọn trước trong khoảng giá trị phù hợp hoặc thậm chí không thể xấp xỉ.



Hình 6. Thời gian tính toán của phương pháp Dombi cải tiến

Ngoài ra, phương pháp Dombi và Dombi cải tiến tự động xác định chính xác các giá trị của hệ số mũ trong khi phương pháp collocation đòi hỏi phải lựa chọn trước miền giá trị của hệ số mũ cũng như bước chia đủ nhỏ của hệ số mũ để có thể đạt được một xấp xỉ với mức độ chính xác cần thiết. Trong nhiều trường hợp, việc lựa chọn trước giá trị của các hệ số mũ là rất khó khăn và gần như bất khả thi (Barrietos và cộng sự, 2018). Đơn cử như trường hợp ví dụ ở Mục 3.1 và 3.2, mặc dù dạng hàm đầu vào là rất đơn giản nhưng sai số của hàm xấp xỉ gần như không giảm theo sự gia tăng của số đoạn chia khi mà khoảng giá trị của các hệ số mũ không được chọn trước một cách phù hợp với giá trị chính xác của nó. Trong thực tế, giá trị phù hợp của các hệ số mũ là chưa biết trước, vì vậy phương pháp Dombi và Dombi cải tiến ưu việt hơn phương pháp collocation.

4. Kết luận

Từ các ví dụ, chúng ta có thể thấy rằng, phương pháp hồi quy phi tuyến chỉ có khả năng xấp xỉ tốt một hàm số bằng tổng các hàm số mũ khi khoảng khảo sát hẹp. Với một khoảng khảo sát rộng, phương pháp này thường thất bại trong việc xấp xỉ.

Đối với phương pháp collocation, việc lựa chọn các giá trị của hệ số mũ có tác động rất lớn đến các độ chính xác của quá trình xấp xỉ. Nếu việc lựa chọn các hệ số mũ không tốt thì quá trình

xấp xỉ sẽ có sai số lớn hoặc thậm chí thất bại.

Đối với phương pháp Dombi và Dombi cải tiến, kết quả xấp xỉ từ các hàm cơ bản đến hàm đạo của vật liệu bê tông nhựa đều đạt độ chính xác cao. Điểm hạn chế của phương pháp là tốn nhiều thời gian tính toán hơn các phương pháp còn lại. Tuy nhiên, tổng thời gian cần thiết để xấp xỉ một hàm đạo của bê tông nhựa là không quá lớn, chỉ dao động trong khoảng từ 2 đến 9 phút tương ứng mức độ sai số tương đối từ 10^{-4} đến 10^{-6} . Khoảng thời gian này là hoàn toàn có thể chấp nhận được để đạt được độ chính xác cao của phép xấp xỉ. Điều này giúp tiết kiệm đáng kể chi phí tính toán khi phân tích kết cấu đàn hồi nhớt bằng phương pháp số.

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Tập 2. Cơ học thủy khí, Cơ học tính toán,...**

Hội Cơ học Việt Nam

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Giám đốc, Tổng biên tập
PHẠM THỊ HIẾU

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HỘI NGHỊ CƠ HỌC TOÀN QUỐC
KỶ NIỆM 45 NĂM THÀNH LẬP VIỆN CƠ HỌC

HÀ NỘI, 09/04/2024

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KỶ NIỆM 45 NĂM THÀNH LẬP VIỆN CƠ HỌC



NHÀ XUẤT BẢN
KHOA HỌC TỰ NHIÊN VÀ CÔNG NGHỆ



NHÀ XUẤT BẢN KHOA HỌC TỰ NHIÊN VÀ CÔNG NGHỆ

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Evaluating the accuracy of linear viscoelastic properties in the time domain analytically approximated from complex modulus data

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Abstract: Currently, asphalt concrete is widely used for pavement in Vietnam owing to the easiness of construction and maintenance and the smoothness for vehicles participating in traffic. However, evaluating the mechanical properties of asphalt concrete is quite complicated. Normally, the characterization of the linear viscoelastic behavior of asphalt concretes is performed in the frequency domain to prevent the effects of viscoplastic deformation. Therefore, converting the LVE characteristics of asphalt concrete materials from the frequency domain to those in the time domain is necessary for pavement analysis in the time domain. The aim of this article is to assess the accuracy of analytical approximate methods used for time-frequency interconversion.

Keywords: linear viscoelastic, relaxation modulus, creep compliance, asphalt mixtures, convolutional integral.

1. Introduction

Relaxation modulus (RM) and creep compliance (CC) are quantities representing the linear viscoelastic (LVE) characteristics of asphalt concrete materials in the time domain. Meanwhile, in the frequency domain, the LVE behavior is described by either the complex modulus or complex compliance. It has been demonstrated that determining the LVE properties of asphalt concrete in the frequency domain is easier to perform than creep or creep testing. The reason is that beyond some temperature levels, permanent deformation takes place, causing difficulties to the measurement of pure LVE strains. According to [1], the irreversible deformation or viscoelastic deformation of asphalt concrete in the creep-recovery test can account for up to 80% of the total deformation. The deformation of asphalt mixtures samples measured in creep or creep-recovery tests at high temperatures certainly include both viscoelastic and viscoplastic deformation. Thus, the calculation of LVE characteristics based on these measurement results will lead to large errors.

In another study, Kim et al. [2] have showed that the RM and CC obtained from creep and creep tests did not coincide with these quantities interconverted from complex modulus data. In addition, after making a comparison between the RM computed from creep test data with the complex modulus data, Zhang et al. [3] have showed that the relative errors between them were up to 30%. Therefore, in most standards and studies, the upper limit of the evaluating temperature during creep testing is set to be lower than 30°C to prevent the influence of permanent deformation on measurement data [4-6].

Many methods have been proposed for converting the RM and CC function from complex modulus data including numerical and analytical methods (see [6] and references quoted therein). Among those, the analytical approximate method proposed by Schapery and Park [6] are widely used in the community for its simplicity and efficiency. According to Emri et al. [7] and Phan et al. [8], the relative errors of the interconverted RM and CC using the method in [6] are at the level of 5%, which is good enough for use in pavement engineering. However, exact values of the RM and CC functions do not exist yet must be numerically computed. Therefore, the aim of this study is to assess the accuracy of the RM and CC via the convolutional integral whose exact values are known a priori.

2. Brief overview of linear viscoelastic theory

The behavior of a linear viscoelastic (LVE) material depends on time (or frequency) and temperature; thus, the relationship between the stress and strain of a LVE material is more complex, represented by LVE functions in time or frequency. Let $\sigma(t)$ and $\varepsilon(t)$ be the stress and strain at a point and at a time t . Considering an excitation of the strain function: $\varepsilon(t) = H(t)$ wherein $H(t)$ is the step function such that $H(t) = 0$ for $t \leq 0$ and $H(t) = 1$ for $t > 0$, the stress response is a decreasing function $\sigma(t) = E(t)$ as illustrated in Fig. 1a. Similarly, when considering a step excitation of stress $\sigma(t) = H(t)$, the strain response is an increasing function $\varepsilon(t) = D(t)$ as illustrated in Fig. 1b. The functions $E(t)$ and $D(t)$ are respectively called the relaxation modulus (RM) and creep compliance (CC) functions, representing the LVE properties of the material in the time domain.

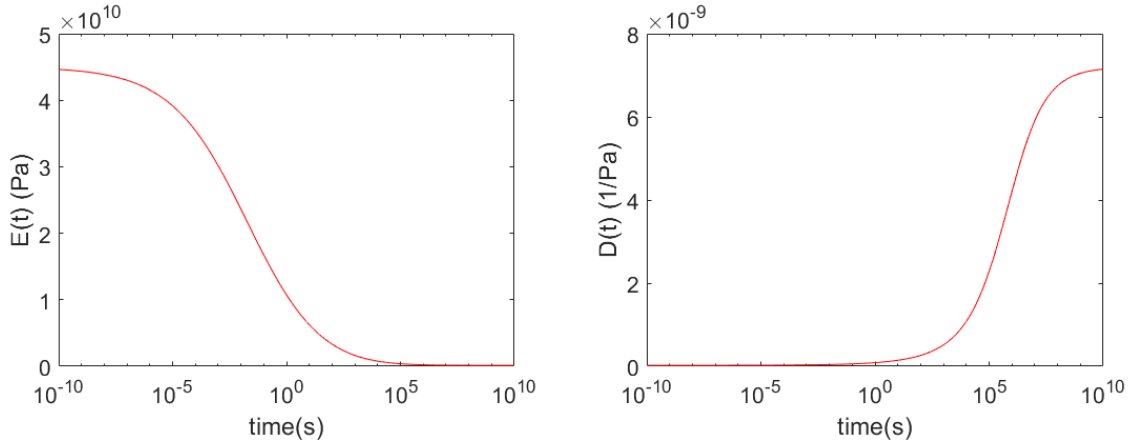


Figure 1. Illustration of relaxation modulus function (a) and creep compliance function (b)

For a LVE material, the stress and strain responses caused by arbitrary excitations of strain and stress can be obtained by applying the Boltzmann superposition principle and are written under convolutional integrals [9-10]:

$$\sigma(t) = \int_{-\infty}^t E(t-\tau) \dot{\varepsilon}(\tau) d\tau; \quad \varepsilon(t) = \int_{-\infty}^t D(t-\tau) \dot{\sigma}(\tau) d\tau \quad (1)$$

For asphalt mixtures, experimental testing in the time domain is difficult because of the existence of permanent strain preventing the accurate capture of pure LVE strain. Therefore, testing in the frequency domain is much preferred in the community. Let us consider a sinusoidal excitation of strain:

$$\varepsilon(t) = \varepsilon_0 \sin(\omega t), \quad (2)$$

where ε_0 is the oscillation amplitude and ω is the angular frequency. Substituting Eq. (2) into Eq. (1) gives rise to:

$$\sigma(t) = \varepsilon_0 \omega \int_{-\infty}^t E(t-\tau) \cos(\omega \tau) d\tau. \quad (3)$$

We also understand that the strain input is the real part of the complex strain $\varepsilon(i\omega t) = \varepsilon_0 (\cos(\omega t) + i \sin(\omega t)) = \varepsilon_0 e^{i\omega t}$. Therefore, formula (3) is rewritten as follows:

$$\sigma(t) = \Re \left[\varepsilon_0 i \omega \int_{-\infty}^t E(t-\tau) e^{i \omega \tau} d\tau \right], \quad (4)$$

where $i^2 = -1$ is the imaginary unit; $\Re[A]$ real part of A . Performing a change of variable $u = t - \tau$ in Eq. (4) give rise to

$$\sigma(t) = \Re \left[\varepsilon_0 e^{i \omega t} \cdot i \omega \int_0^{\infty} E(u) e^{-i \omega u} du \right]. \quad (5)$$

In addition, the complex stress is also calculated as follows:

$$\sigma(t) = \Re[\sigma(i \omega t)] = \Re[\varepsilon(i \omega t) \cdot E^*(i \omega)]. \quad (6)$$

By comparing Eq. (6) with Eq. (5), and making use of $\varepsilon(i \omega t) = \varepsilon_0 \cdot e^{i \omega t}$, one can derive the complex modules [9]:

$$E^*(i \omega) = i \omega \int_0^{\infty} E(t) e^{-i \omega t} dt. \quad (7)$$

It can be demonstrated that [10]:

$$\sigma(t) = \varepsilon_0 \omega \cos \omega t \int_0^{+\infty} E(u) \cos \omega u du + \varepsilon_0 \omega \sin \omega t \int_0^{+\infty} E(u) \sin \omega u du, \quad (8)$$

or

$$\sigma(t) = E'(\omega) \varepsilon(t) + \frac{E''(\omega)}{\omega} \dot{\varepsilon}(t), \quad (9)$$

where: $E'(\omega) = \omega \int_0^{\infty} E(u) \sin \omega u du = \omega \int_0^{\infty} E(t) \sin \omega t dt$ is the storage modulus, representing elastic

behavior and $E''(\omega) = \omega \int_0^{\infty} E(u) \cos \omega u du = \omega \int_0^{\infty} E(t) \cos \omega t dt$ is the loss modulus, representing the

viscous behavior. The storage and loss modules are in fact two components of the complex modulus whose norm and argument are respectively called the dynamic modulus and phase angle [9-10]:

$$|E^*(\omega)| = \sqrt{E'(\omega)^2 + E''(\omega)^2}, \quad \varphi_E(\omega) = \tan^{-1} \frac{E''(\omega)}{E'(\omega)}. \quad (10)$$

It was demonstrated that the storage and loss modulus are not independent yet interrelated with each other via Kramers-Kronig relationships [9-10]:

$$E'(\omega) = \frac{2\omega^2}{\pi} \int_0^{\infty} \frac{E''(w)/w}{\omega^2 - w^2} dw, \quad E''(\omega) = \frac{2\omega}{\pi} \int_0^{\infty} \frac{E'(w)}{w^2 - \omega^2} dw, \quad (11)$$

$$\int_0^{+\infty} \frac{E''(\omega) \cos \omega t}{\omega} d\omega = \int_0^{+\infty} \frac{E'(\omega) \sin \omega t}{\omega} d\omega, \quad \forall t \geq 0 \quad (12)$$

Similarly, the relationship between the dynamic modulus and phase angles can also be derived:

$$\varphi_E(\omega) = \frac{1}{\pi} \int_0^{+\infty} \ln \left| \frac{u + \omega}{u - \omega} \right| \frac{d \ln |E^*(u)|}{d \ln u} d \ln u. \quad (13)$$

Because the properties of the material are unique, quantities that characterize the properties of the material such as relaxation modulus $E(t)$, creep compliance $D(t)$, complex modulus $E^*(\omega)$ and

complex compliance $D^*(\omega)$ can be converted into each other. The LVE characteristics in frequency and time domain are interrelated through Carson transformations [6,9]:

$$\hat{E}(s) = s \int_0^\infty E(t) e^{-st} dt; \hat{D}(s) = s \int_0^\infty D(t) e^{-st} dt; \hat{E}(s) \cdot \hat{D}(s) = 1 \quad (14)$$

$$E^*(\omega) = \hat{E}(s) \Big|_{s \rightarrow i\omega}; D^*(\omega) = \hat{D}(s) \Big|_{s \rightarrow i\omega} \quad (15)$$

In practice, the approximate analytical formulas of Schapery and Park [6] is widely used for its efficiency and simplicity, which is summarized in Eqs. (16)-(17):

$$E\left(t = \frac{1}{\omega}\right) = \frac{E'(\omega)}{\lambda'(\omega)}; \lambda'(\omega) = \Gamma(1-n) \cos\left(n \frac{\pi}{2}\right) \quad (16)$$

$$E(t) \cdot D(t) = \frac{\sin(n\pi)}{n\pi} \quad (17)$$

where $E'(\omega)$ is storage modulus, $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ is Gamma function and n is the local slope of the curve of $\log(E'(\omega))$ versus $\log(\omega)$.

The relationship between the relaxation modulus $E(t)$ and the creep compliance $D(t)$ is given in [9-10]:

$$I(t) = \int_0^t E(t-\tau) \cdot D(\tau) \cdot d\tau = \int_0^t E(\tau) \cdot D(t-\tau) \cdot d\tau = t. \quad (18)$$

In addition, the complex modulus is the inverse of the complex compliance and vice-versa [9-10]:

$$E^*(\omega) D^*(\omega) = 1. \quad (19)$$

3. Numerical results

In this example, the purpose is to evaluate the accuracy of the relaxation modulus and creep compliance function analytically derived from dynamic modulus data using the Schapery and Park's formulas given in Eq. (16) while computing the convolutional integral in Eq. (18). Therefore, the results of LVE characterization of the asphalt mixtures in [11] were reused here. The stress-strain relationships in the frequency domain of the materials were represented by the well-known Huet – Sayegh (HS) model as described in Fig. 2.

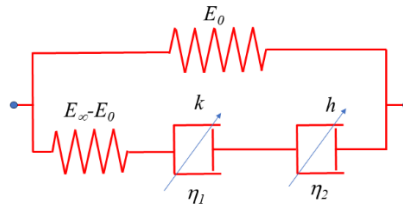


Figure 2. Illustration of Huet-Sayegh model

The complex modulus of HS model was given in [12] and is reminded in Eq. (20):

$$E_{HS}^*(\omega) = E_0 + \frac{E_\infty - E_0}{1 + \delta(i\omega\tau)^{-k} + (i\omega\tau)^{-h}} \quad (20)$$

The HS parameters of the materials were determined in [11] by minimizing the errors between the theoretical and experimental data and are summarized in Table 1.

Table 1. Huet-Sayegh parameters of the studied mixtures.

Mixture	$E_o(MPa)$	$E_\infty(MPa)$	k	h	δ	$\tau(s)$	$T_{ref}(^{\circ}C)$
Modified	139	45,000	0.250	0.500	4.938	16.302	20
Unmodified	140	19,550	0.390	0.500	12.000	20.000	20

Based on the parameters in Table 1 and formulas in Eq. (16)-(17), we calculated the RM and CC and are shown in Figs. 1(a) and 1(b).

Since the HS model does not exist exact formula of RM and CC in time domain, the convolutional integral in Eq. (18) was used in this study to assess the accuracy of the RM and CC. The exact values of the integral are known a priori; thus, the errors of the integral can be calculated as:

$$err = [I(t) - t] / t \quad (21)$$

In practice, the RM and CC functions are decomposed into sum of exponential functions (SEFs), called the Prony series to derived recursive formulas between stress and strain [13]. As a result, the computational cost can be significantly reduced [14]. By using the modified Dombi algorithm [14], the RM and CC functions were decomposed into SEFs with high accuracy:

$$E(t) = E_\infty + \sum_{i=1}^{n_i} a_i e^{\alpha_i t}, \quad (22)$$

$$D(t) = D_0 - \sum_{j=1}^{n_j} b_j e^{\beta_j t}. \quad (23)$$

As result, the convolutional integral in Eq. (18) can be analytically calculated by:

$$\int_0^t E(t-\tau)D(\tau)d\tau = \begin{cases} t + E_0 \sum_{j=1}^n \frac{b_j}{\beta_j} (1 - e^{\beta_j t}) - D_0 \sum_{i=1}^m \frac{a_i}{\alpha_i} (1 - e^{\alpha_i t}) \\ - \sum_{i=1}^m \sum_{j=1}^n \frac{a_i b_j}{\beta_j - \alpha_i} (e^{\beta_j t} - e^{\alpha_i t}) \text{ if } \alpha_i \neq \beta_j \\ t + E_0 \sum_{j=1}^n \frac{b_j}{\beta_j} (1 - e^{\beta_j t}) - D_0 \sum_{i=1}^m \frac{a_i}{\alpha_i} (1 - e^{\alpha_i t}) \\ - \sum_{i=1}^m \sum_{j=1}^n a_i b_j e^{\alpha_i t} \cdot t \text{ if } \alpha_i = \beta_j \end{cases} \quad (24)$$

Furthermore, the integral was also numerically calculated using the mid-point rules:

$$I(t) = \int_0^{t_1} E(t-\tau)D(\tau)d\tau + \int_{t_1}^{t_2} E(t-\tau)D(\tau)d\tau + \dots + \int_{t_{n-1}}^{t_n} E(t-\tau)D(\tau)d\tau \quad (25)$$

$$\int_{t_{m-1}}^{t_m} E(t-\tau)D(\tau)d\tau \approx \frac{1}{2} [(E(t-t_m) + E(t-t_{m-1})) \times \Delta t] D\left(t_{m-1} + \frac{\Delta t}{2}\right), \quad m=1..n \quad (26)$$

$$\Delta t = t_m - t_{m-1} \quad (27)$$

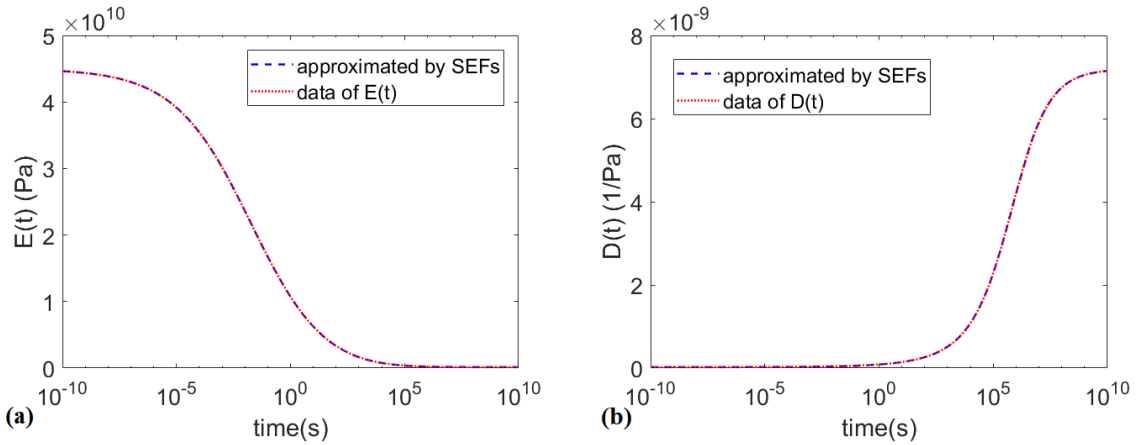


Figure 3. Decomposition of RM (a) and CC (b) functions into SEFs

Fig. 3 shows the results of the decomposition of RM and CC functions into SEFs. It can be seen that the source functions and the SEFs coincide very well with each other. In fact, the relative errors of the decomposition were less than 0.1% at all evaluated points (see Fig. 4(a) and (b)).

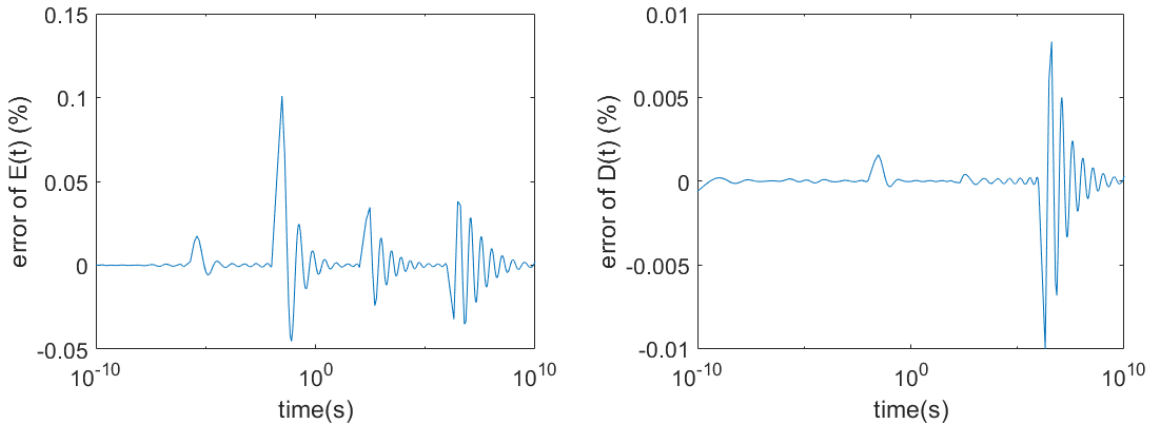


Figure 4. Errors of the decomposition of RM (a) and CC (b) functions into SEFs

Figs. 5 and 6 display the results of calculating the convolutional integral of the unmodified asphalt concrete materials, while Figs. 7 and 8 show the results obtained with the modified asphalt mixture. The graphs respectively show the errors when using the analytical and numerical integration methods. Based on the charts in Figs. 5 and 6, it can be seen that when using the explicit integration method, there are larger errors than when using the numerical integration method, especially in the early period from 10^{-10} s to 10^{-8} where significant errors can be clearly observed. Meanwhile, the error when using numerical integration is relatively small. According to the obtained results, the relative errors of the convolutional integrals ranged from -3% to 5%, which is in good accord with previous findings of Emri et al. [7] and Phan et al. [8].

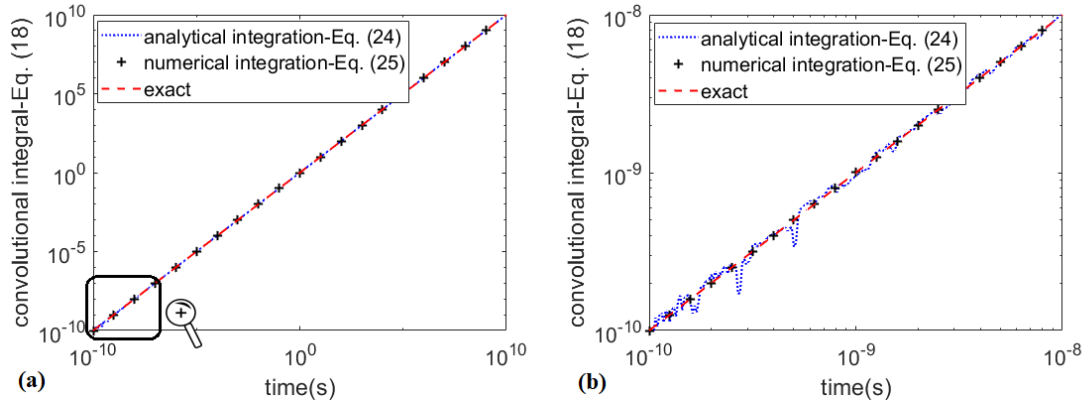


Figure 5. Values of the convolutional integral computed with the HS parameters of the unmodified mixture (a) and zoom in the time range from 10^{-10} to 10^{-8} s (b)

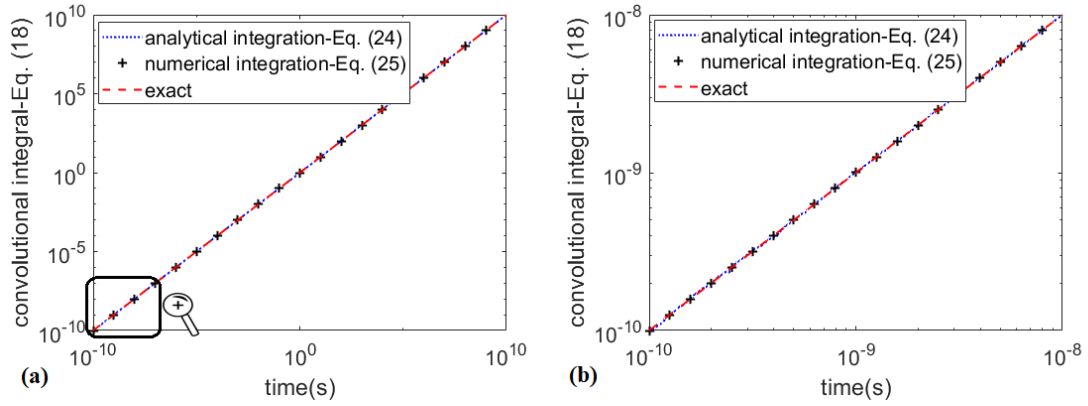


Figure 6. Values of the convolutional integral computed with the HS parameters of the modified mixture (a) and zoom in the time range from 10^{-10} to 10^{-8} s (b)

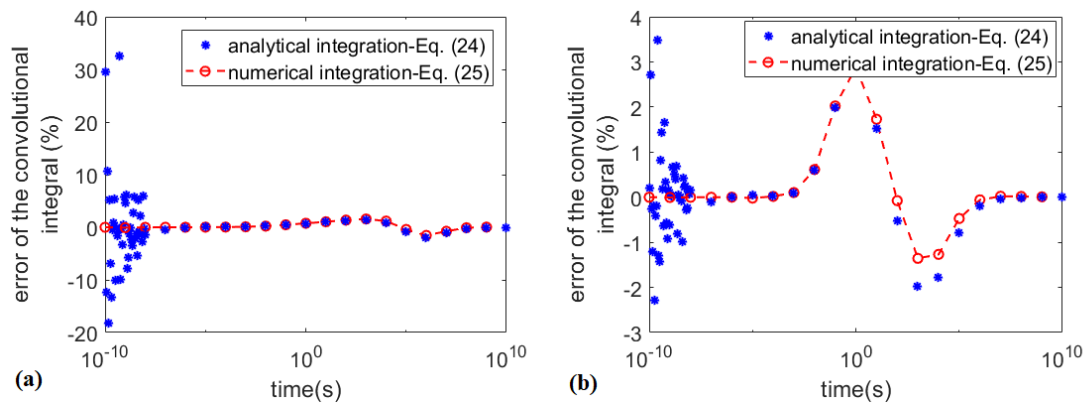


Figure 7. Errors of the convolutional integrals obtained with (a) unmodified mixture, (b) modified mixture

4. Conclusion

The accuracy of the relaxation modulus and creep compliance function analytically approximated from complex modulus data has been successfully assessed by evaluating the

convolutional integral. Owing to the exact value of the convolutional integral, the relative errors of the converted relaxation modulus and creep compliance were evaluated in a reliable manner. The results of this work again confirm the level of accuracy of the converted relaxation modulus and creep compliance (approximately 5%) already discussed in previous studies. The extent of errors is small enough for uses in analysis and design of asphalt pavement.

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